基于积分本构黏弹性带的横向非线性动力学[1)]

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摘要　研究了黏弹性传动带在 1:1 内共振时的横向非平面非线性动力学特性．首先，利用 Hamilton 原理建立了黏弹性传动带横向非平面非线性动力学方程．然后综合应用多尺度法和 Galerkin 离散法对偏微分形式的动力学方程进行摄动分析，得到了四维平均方程．对平均方程的稳定性进行了分析，从理论上讨论了动力系统解的稳定性变化情况．最后数值模拟结果表明黏弹性传动带系统存在混沌运动、概周期运动和周期运动．

关键词　黏弹性传动带，非线性，横向振动，内共振，混沌

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引　言

黏弹性传动带系统在实际工程中应用广泛．带传动系统是机械装置动力传动和变速的主要部件，具有结构简单、传动方便、不受距离限制、容易调节更换等特点．尽管这些设备有许多优点，但其在高速运转时，将会产生很大的横向振动，对应用有许多不利方面．随着科学技术的迅猛发展，工程应用的精度要求越来越高，线性理论分析已经很难满足要求．由于非线性因素的影响，黏弹性传动带系统的动力学行为表现也更为复杂，呈现出分岔和混沌的现象．从而，有必要研究传动带系统的非线性动力学行为，揭示非线性对系统动力学行为的影响．从工程技术的角度来说，其任务是为减小系统振动或有效利用振动使系统具有合理的结构形式和参数．因此，对其进行研究具有重要的理论意义和应用价值．

动力传动带是种常见工程元件，忽略其弯曲刚度都可以模型化为轴向运动弦线进行研究．其动力学问题引起了国内外学者的极大关注． Wickert 和 Mote[1] 给出了关于轴向运动材料的振动和稳定性研究工作的综述． Zhang 和 Zu[2,3] 研究了轴向运动传动带的非线性自由振动和强迫振动． Chen 和 Zu[4,5] 研究了轴向运动黏弹性带的参数振动和动力学响应，黏弹性带为满足 Boltzmann 叠加原理的黏弹性材料．Pakdemirli 等[6] 应用 Hamilton 原理建立了轴向速度周期变化时弦线的振动动力学方程，采用 Glalerkin 方法将偏微分方程截断为常微分方程，然后分析了系统的稳定性． Zhang 等[7] 利用多尺度法和 Galerkin 离散方法，分析了 1:3 内共振和主参数共振情况下的黏弹性带的周期运动和混沌动力学． Zhang 等[8] 利用多尺度法和 Galerkin 离散方法，研究了黏弹性传动带系统的多自由度非线性振动特性．陈树辉等[9] 研究了轴向运动梁的非线性内共振． Chen 等[10] 对黏弹性轴向运动弦线的分岔与混沌情况进行了研究．

关于轴向运动弦线非线性动力学的研究大多仅限于平面内，其空间非线性动力学研究则较少．早期 Shih[11,12] 对轴向运动弦线三维非线性振动问题进行了研究． Huang 等[13] 研究了轴向运动弹性弦线三维非线性振动由于轴向张力周期涨落导致参数振动的稳定性，并得到非共振和组合共振时不稳定条件．Chen 等[14] 利用 Hamilton 原理建立了黏弹性传动带的空间非线性振动方程． Zhang 等[15] 研究了轴向运动黏弹性带的周期和混沌动力学响应，采用 Kelvin 微分本构模型描述黏弹性带的材料特性．

本文研究了参数激励作用下黏弹性传动带在 1:1 内共振时的横向非线性动力学行为．首先建立了具有线性外阻尼情况下的黏弹性传动带非平面横向振动的动力学方程，黏弹性材料采用积分本构模型来

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描述. 然后利用多尺度法和 Galerkin 离散法得到了黏弹性传动带系统在 1:1 内共振和主参数共振时直角坐标形式的平均方程. 对平均方程的定常解进行了稳定性分析. 最后利用数值模拟方法分析了黏弹性传动带系统的非线性动力学行为, 得到了系统在不同参数下的混沌运动、概周期运动、两倍周期运动和周期运动.

1 动力学方程

在研究中, 首先需要建立带的动力学模型如图 1 所示. 在分析过程中, 采用固定的直角坐标系 $oxyz$ 描述. 其中, u,v,w 分别为带上任意一点在 x,y,z 方向的位移, 两个简支端之间的距离为 L, c_0 为传动带的轴向运动速度, $\boldsymbol{P}$ 为带的张力, 由于考虑稳态张力有一个周期扰动的情况, 即可以表示为 $\boldsymbol{P}=P_0+P_1\cos\Omega t$.

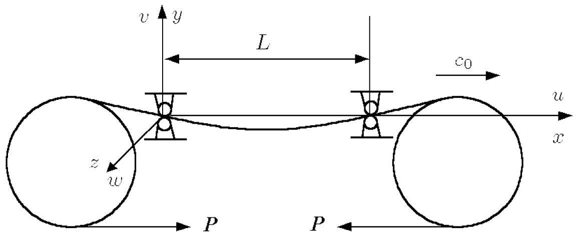

图 1 传动带系统的动力学模型

Fig.1 The model of an axially moving belt

由于通常情况下黏弹性运动带主要发生横向振动, 因此本文只考虑横向振动的情形, 同时考虑其大变形, 应变具体表达式为

$$\varepsilon=\frac{1}{2}(v_{,x}^2+w_{,x}^2) \tag{1}$$

采用积分型本构模型描述材料的黏弹性特性. 积分型本构模型由 Boltzmann 叠加原理导出, 具体表述如下

$$\sigma(x,t)=\varepsilon(x,t)E_0+\int_0^t \dot{E}(t-s)\varepsilon(x,s)\mathrm{d}s \tag{2}$$

其中, $E(t)$ 是张力松弛函数, E_0 是在 $t=0$ 时 $E(t)$ 的值.

运用 Hamilton 原理建立了黏弹性传动带的横向非线性动力学方程.

$$\begin{aligned}
&\rho v_{,tt}+2\rho c_0 v_{,tx}+\rho c_0^2 v_{,xx}=\frac{3}{2}E_0(v_{,x})^2 v_{,xx}+\\
&\quad E_0 w_{,x}w_{,xx}v_{,x}+\frac{1}{2}E_0(w_{,x})^2 v_{,xx}+\\
&\quad v_{,xx}\int_0^t \dot{E}\frac{1}{2}[(v_{,x})^2+(w_{,x})^2]\mathrm{d}\tau+\\
&\quad v_{,x}\int_0^t \dot{E}(v_{,x}v_{,xx}+w_{,x}w_{,xx})\mathrm{d}\tau+\\
&\quad \frac{P_0+P_1\cos\Omega t}{A}v_{,xx}-cv_{,t}
\end{aligned} \tag{3a}$$

$$\begin{aligned}
&\rho w_{,tt}+2\rho c_0 w_{,tx}+\rho c_0^2 w_{,xx}=\frac{3}{2}E_0 w_x^2 w_{,xx}+\\
&\quad E_0 v_{,x}v_{,xx}w_{,x}+\frac{1}{2}E_0(v_{,x})^2 w_{,xx}+\\
&\quad w_{,xx}\int_0^t \dot{E}\frac{1}{2}[(v_{,x})^2+(w_{,x})^2]\mathrm{d}\tau+\\
&\quad w_{,x}\int_0^t \dot{E}(v_{,x}v_{,xx}+w_{,x}w_{,xx})\mathrm{d}\tau+\\
&\quad \frac{P_0+P_1\cos\Omega t}{A}w_{,xx}-cw_{,t}
\end{aligned} \tag{3b}$$

其中 ρ 为带的密度, A 为带的横截面, c 为外阻尼系数.

引入如下无量纲变量

$$\left.\begin{aligned}
&w^*=\frac{w}{L},\quad v^*=\frac{v}{L},\quad x^*=\frac{x}{L},\quad a=\frac{P_1}{P_0},\\
&t^*=t\sqrt{\frac{P_0}{\rho AL^2}},\ \gamma=c_0\sqrt{\frac{\rho A}{P_0}},\ E_e=\frac{E_0 A}{P_0},\\
&\omega=\Omega\sqrt{\frac{\rho AL^2}{P_0}},\ \dot{E}^*=\dot{E}(t-\tau)\frac{A}{P_0},\ \mu=\frac{1}{2}c\sqrt{\frac{A}{P_0\rho}}
\end{aligned}\right\} \tag{4}$$

将上述变换式 (4) 代入式 (3), 化简并去掉 "*" 可得到黏弹性传动带无量纲形式的非线性动力学方程

$$\begin{aligned}
&v_{,tt}+2\gamma v_{,tx}+(\gamma^2-1-a\cos\omega t)v_{,xx}+\\
&\quad 2\mu v_{,t}-N_1(w,v)=0
\end{aligned} \tag{5a}$$

$$\begin{aligned}
&w_{,tt}+2\gamma w_{,tx}+(\gamma^2-1-a\cos\omega t)w_{,xx}+\\
&\quad 2\mu w_{,t}-N_2(w,v)=0
\end{aligned} \tag{5b}$$

式中

$$\begin{aligned}
&N_1(w,v)=\frac{3}{2}E_e(v_{,x})^2 v_{,xx}+E_e w_{,x}w_{,xx}v_{,x}+\\
&\quad v_{,xx}\int_0^t\left(\frac{1}{2}(v_{,x})^2+\frac{1}{2}(w_{,x})^2\right)\frac{\partial E}{\partial\tau}\mathrm{d}\tau+\\
&\quad v_{,x}\int_0^t\frac{\partial E}{\partial\tau}(v_{,x}v_{,xx}+w_{,x}w_{,xx})\mathrm{d}\tau+\\
&\quad \frac{1}{2}E_e(w_{,x})^2 v_{,xx}
\end{aligned} \tag{6a}$$

$$N_2(w,v)=\frac{3}{2}E_e(w_{,x})^2w_{,xx}+E_ev_{,x}v_{,xx}w_{,x}+ \\ w_{,xx}\int_0^t\left[\frac{1}{2}(v_{,x})^2+\frac{1}{2}(w_{,x})^2\right]\frac{\partial E}{\partial\tau}\mathrm{d}\tau+ \\ w_{,x}\int_0^t\frac{\partial E}{\partial\tau}(v_{,x}v_{,xx}+w_{,x}w_{,xx})\mathrm{d}\tau+ \\ \frac{1}{2}E_e(v_{,x})^2w_{,xx} \tag{6b}$$

2 摄动分析

为了便于研究，引入质量、陀螺和线性刚度算子如下

$$M=\boldsymbol{I},\quad G=2\gamma\frac{\partial}{\partial x},\quad K=(\gamma^2-1)\frac{\partial^2}{\partial x^2} \tag{7}$$

其中 $\boldsymbol{I}$ 为单位矩阵，G 和 K 为偏微分算子.

将方程 (7) 代入方程 (5), 并引入小参数 ε[16], 则方程 (5) 可以写为带有弱非线性项和参数激励项的连续系统，如下

$$Mv_{,tt}+Gv_{,t}+Kv=\varepsilon N_1(w,\ v)-2\varepsilon\mu\frac{\partial v}{\partial t}+ \\ \varepsilon a\cos\omega t\frac{\partial^2 v}{\partial x^2} \tag{8a}$$

$$Mw_{,tt}+Gw_{,t}+Kw=\varepsilon N_2(w,v)-2\varepsilon\mu\frac{\partial w}{\partial t}+ \\ \varepsilon a\cos\omega t\frac{\partial^2 w}{\partial x^2} \tag{8b}$$

采用多尺度法 [16,17] 对偏微分方程进行摄动分析，设方程 (8) 的一致渐进解为

$$v(x,t,\varepsilon)=v_0(x,T_0,T_1)+\varepsilon v_1(x,T_0,T_1)+\cdots \tag{9a}$$

$$w(x,t,\varepsilon)=w_0(x,T_0,T_1)+\varepsilon w_1(x,T_0,T_1)+\cdots \tag{9b}$$

式中 $T_0=t$, $T_1=\varepsilon t$.

微分算子为

$$\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\partial}{\partial T_0}\frac{\partial T_0}{\partial t}+\frac{\partial}{\partial T_1}\frac{\partial T_1}{\partial t}+\cdots= \\ D_0+\varepsilon D_1+\cdots \tag{10}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}=(D_0+\varepsilon D_1+\cdots)^2= \\ D_0^2+2\varepsilon D_0D_1+\cdots \tag{11}$$

式中 $D_n=\partial/\partial T_n$, $n=0,1$.

将方程 (9)～ 方程 (11) 代入方程 (8), 令等式两端 ε 同次幂的系数相等，得到

ε^0 阶项

$$M\frac{\partial^2 v_0}{\partial T_0^2}+G\frac{\partial v_0}{\partial T_0}+Kv_0=0 \tag{12a}$$

$$M\frac{\partial^2 w_0}{\partial T_0^2}+G\frac{\partial w_0}{\partial T_0}+Kw_0=0 \tag{12b}$$

ε^1 阶项

$$M\frac{\partial^2 v_1}{\partial T_0^2}+G\frac{\partial v_1}{\partial T_0}+Kv_1=N_1(w_0,v_0)-G\frac{\partial v_0}{\partial T_1}- \\ 2M\frac{\partial^2 v_0}{\partial T_0\partial T_1}-2\mu\frac{\partial v_0}{\partial T_0}+a\cos\omega T_0\frac{\partial^2 v_0}{\partial x^2} \tag{13a}$$

$$M\frac{\partial^2 w_1}{\partial T_0^2}+G\frac{\partial w_1}{\partial T_0}+Kw_1=N_2(w_0,v_0)-G\frac{\partial w_0}{\partial T_1}- \\ 2M\frac{\partial^2 w_0}{\partial T_0\partial T_1}-2\mu\frac{\partial w_0}{\partial T_0}+a\cos\omega T_0\frac{\partial^2 w_0}{\partial x^2} \tag{13b}$$

式中

$$N_1(w_0,v_0)=\frac{3}{2}E_e\Big(\frac{\partial v_0}{\partial x}\Big)^2\frac{\partial^2 v_0}{\partial x^2}+ \\ E_e\frac{\partial w_0}{\partial x}\frac{\partial w_0^2}{\partial x^2}\frac{\partial v_0}{\partial x}+\frac{1}{2}E_e\Big(\frac{\partial w_0}{\partial x}\Big)^2\frac{\partial^2 v_0}{\partial x^2}+ \\ \frac{\partial^2 v_0}{\partial x^2}\int_0^t\left(\frac{1}{2}\Big(\frac{\partial v_0}{\partial x}\Big)^2+\frac{1}{2}\Big(\frac{\partial w_0}{\partial x}\Big)^2\right)\frac{\partial E}{\partial\tau}\mathrm{d}\tau+ \\ \frac{\partial v_0}{\partial x}\int_0^t\frac{\partial E}{\partial\tau}\Big(\frac{\partial v_0}{\partial x}\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial w_0^2}{\partial x^2}\Big)\mathrm{d}\tau \tag{14a}$$

$$N_2(w_0,v_0)=\frac{3}{2}E_e\Big(\frac{\partial w_0}{\partial x}\Big)^2\frac{\partial^2 w_0}{\partial x^2}+ \\ E_e\frac{\partial v_0}{\partial x}\frac{\partial v_0^2}{\partial x^2}\frac{\partial w_0}{\partial x}+\frac{1}{2}E_e\Big(\frac{\partial v_0}{\partial x}\Big)^2\frac{\partial^2 w_0}{\partial x^2}+ \\ \frac{\partial^2 w_0}{\partial x^2}\int_0^t\left(\frac{1}{2}\Big(\frac{\partial v_0}{\partial x}\Big)^2+\frac{1}{2}\Big(\frac{\partial w_0}{\partial x}\Big)^2\right)\frac{\partial E}{\partial\tau}\mathrm{d}\tau+ \\ \frac{\partial w_0}{\partial x}\int_0^t\frac{\partial E}{\partial\tau}\Big(\frac{\partial v_0}{\partial x}\frac{\partial^2 v_0}{\partial x^2}+\frac{\partial w_0}{\partial x}\frac{\partial w_0^2}{\partial x^2}\Big)\mathrm{d}\tau \tag{14b}$$

下面引进模态函数对系统进行 Galerkin 离散. 由于方程 (12) 是线性偏微分方程，它的通解可以用已经分离变量的复数形式表示为 [18]

$$v_0(T_0,T_1,x)=\psi_l(x)A_l(T_1)\mathrm{e}^{\mathrm{i}\omega_l T_0}+\mathrm{cc} \tag{15a}$$

$$w_0(T_0,T_1,x)=\psi_n(x)A_n(T_1)\mathrm{e}^{\mathrm{i}\omega_n T_0}+\mathrm{cc} \tag{15b}$$

这里模态函数为 $\psi_k=\sqrt{2}\sin(k\pi x)\mathrm{e}^{(\mathrm{i}k\pi\gamma x)}$, $k=n,l$. ω_k 为相应的线性系统的固有频率，$\omega_k=k\pi(1-\gamma^2)$, $k=n,l$, cc 为前项的复共轭项.

由于共振是导致机械系统破坏的主要原因,故研究黏弹性传动带 1:1 内共振和主参数共振的情况,共振关系为

$$\omega_l=\omega_n,\ \omega_l=\frac{1}{2}\omega+\varepsilon\sigma_1,\ \omega_n=\frac{1}{2}\omega+\varepsilon\sigma_2 \tag{16}$$

式中 σ_1 和 σ_2 为两个调协参数.

假设积分型模型的黏弹性材料是一种标准线性固体，其松弛函数表达式如下

$$E(t)=1-b+be^{-\alpha t} \tag{17}$$

其中 b 为黏弹性系数， α 为黏弹性指数.

将方程 (15)∼ 方程 (17) 代入方程 (13), 合并同类项得

$$\begin{aligned}&MD_0^2v_1+GD_0v_1+Kv_1=\Gamma_1(T_1,x)\mathrm{e}^{\mathrm{i}\left(\frac{1}{2}\omega+\varepsilon\sigma_1\right)T_0}-\\&\quad\frac{b\alpha}{\mathrm{i}2\omega_n+\alpha}\left[\frac{1}{2}\bar\psi_l''(\psi_n')^2+\bar\psi_l\psi_n'\psi_n''\right]\cdot\\&\quad\bar A_lA_n^2\mathrm{e}^{\mathrm{i}\left[\frac{1}{2}\omega+\varepsilon(2\sigma_2-\sigma_1)\right]T_0}+\\&\quad E_e\left[\frac{1}{2}\bar\psi_l''(\psi_n')^2+\bar\psi_l'\psi_n'\psi_n''\right]\bar A_lA_n^2\mathrm{e}^{\mathrm{i}\left[\frac{1}{2}\omega+\varepsilon(2\sigma_2-\sigma_1)\right]T_0}+\\&\quad\frac{a}{2}\bar\psi_l''\bar A_l\mathrm{e}^{\mathrm{i}\left(\frac{1}{2}\omega-\varepsilon\sigma_1\right)T_0}+\mathrm{cc}+NST\end{aligned}\tag{18a}$$

$$\begin{aligned}&MD_0^2w_1+GD_0w_1+Kw_1=\Gamma_2(T_1,x)\mathrm{e}^{\mathrm{i}\left(\frac{1}{2}\omega+\varepsilon\sigma_2\right)T_0}-\\&\quad\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}\left[\frac{1}{2}(\psi_l')^2\bar\psi_n''+\bar\psi_n\psi_l'\psi_l''\right]\cdot\\&\quad A_l^2\bar A_n\mathrm{e}^{\mathrm{i}\left[\frac{1}{2}\omega+\varepsilon(2\sigma_1-\sigma_2)\right]T_0}+\\&\quad E_e\left[\frac{1}{2}\bar\psi_n''(\psi_l')^2+\bar\psi_n'\psi_l'\psi_l''\right]A_l^2\bar A_n\mathrm{e}^{\mathrm{i}\left[\frac{1}{2}\omega+\varepsilon(2\sigma_1-\sigma_2)\right]T_0}+\\&\quad\frac{a}{2}\bar\psi_n''\bar A_n\mathrm{e}^{\mathrm{i}\left(\frac{1}{2}\omega-\varepsilon\sigma_2\right)T_0}+\mathrm{cc}+NST\end{aligned}\tag{18b}$$

式中

$$\begin{aligned}&\Gamma_1(T_1,x)=-2\mathrm{i}M\omega_l\psi_l\dot A_l-G\psi_l\dot A_l-2\mathrm{i}\mu\omega_l\psi_lA_l+\\&\quad A_l^2\bar A_lE_e\left[3\psi_l'\bar\psi_l'\psi_l''+\frac{3}{2}(\psi_l')^2\bar\psi_l''\right]+\\&\quad A_lA_n\bar A_nE_e(\psi_l'\psi_n'\bar\psi_n''+\psi_l'\bar\psi_n'\psi_n''+\psi_n'\bar\psi_n'\psi_l'')-\\&\quad A_lA_n\bar A_nb(\psi_l\bar\psi_n'\psi_n''+\psi_l\psi_n'\bar\psi_n''+\psi_l''\psi_n'\bar\psi_n')-\\&\quad b(\psi_l\psi_l'\bar\psi_l''+\psi_l\bar\psi'_l\psi_l''+\psi_l''\psi_l'\bar\psi_l')A_l^2\bar A_l-\\&\quad\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}\left(\bar\psi_l\psi_l'\psi_l''+\frac{1}{2}\bar\psi_l''(\psi_l')^2\right)A_l^2\bar A_l\end{aligned}\tag{19a}$$

$$\begin{aligned}&\Gamma_2(T_1,x)=-2\mathrm{i}M\omega_n\psi_n\dot A_n-G\psi_n\dot A_n-2\mathrm{i}\mu\omega_n\psi_nA_n+\\&\quad E_eA_n^2\bar A_n\left[3\psi_n'\bar\psi_n'\psi_n''+\frac{3}{2}(\psi_n')^2\bar\psi_n''\right]+\\&\quad E_eA_nA_l\bar A_l(\psi_n'\bar\psi_l'\psi_l''+\psi_n'\psi_l'\bar\psi_l''+\psi_l'\bar\psi_l'\psi_n'')-\\&\quad bA_n\bar A_lA_l(\psi_n\psi_l'\bar\psi_l''+\psi_n\bar\psi_l'\psi_l''+\bar\psi_l'\psi_l'\psi_n'')-\\&\quad b(\psi_n''\psi_n'\bar\psi_n'+\psi_n\psi_n'\bar\psi_n''+\psi_n\bar\psi_n'\psi_n'')A_n^2\bar A_n-\\&\quad\frac{b\alpha}{(\mathrm{i}2\omega_n+\alpha)}\left(\frac{1}{2}\bar\psi_n''(\psi_n')^2+\bar\psi_n\psi_n'\psi_n''\right)A_n^2\bar A_n\end{aligned}\tag{19b}$$

这里 “·” 表示对时间 T_0 的导数， “′” 表示对变量 x 的导数， NST 代表不产生长期项的所有项.

因为方程 (18) 相应的齐次方程有非平凡解，所以只有当非齐次方程 (18) 满足可解条件时，方程才有解. 可解性条件要求方程 (18) 的右端与对应的齐次伴随方程的解正交，所以有可解性条件

$$\begin{aligned}&\int_0^1\left[\Gamma_1(T_1,x)+\frac{a}{2}\bar\psi_l''\bar A_l\mathrm{e}^{\mathrm{i}(-2\sigma_1)T_1}\right]\bar\psi_l\mathrm{d}x+\\&\quad\int_0^1E_e\left[\frac{1}{2}\bar\psi_l''(\psi_n')^2+\bar\psi_l'\psi_n'\psi_n''\right]A_n^2\bar A_l\mathrm{e}^{\mathrm{i}2(\sigma_2-\sigma_1)T_1}\bar\psi_l\mathrm{d}x-\\&\quad\int_0^1\frac{b\alpha}{(\mathrm{i}2\omega_n+\alpha)}\frac{1}{2}(\psi_n')^2\bar\psi_l''A_n^2\bar A_l\mathrm{e}^{\mathrm{i}2(\sigma_2-\sigma_1)T_1}\bar\psi_l\mathrm{d}x-\\&\quad\int_0^1\frac{b\alpha}{(\mathrm{i}2\omega_n+\alpha)}\bar\psi_l\psi_n'\psi_n''A_n^2\bar A_l\mathrm{e}^{\mathrm{i}2(\sigma_2-\sigma_1)T_1}\bar\psi_l\mathrm{d}x=0\end{aligned}\tag{20a}$$

$$\begin{aligned}&\int_0^1\left[\Gamma_2(T_1,x)+\frac{a}{2}\bar\psi_n''\bar A_n\mathrm{e}^{\mathrm{i}(-2\sigma_2)T_1}\right]\bar\psi_n\mathrm{d}x+\\&\quad\int_0^1E_e\left[\frac{1}{2}\bar\psi_n''(\psi_l')^2+\bar\psi_n'\psi_l'\psi_l''\right]A_l^2\bar A_n\mathrm{e}^{\mathrm{i}2(\sigma_1-\sigma_2)T_1}\bar\psi_n\mathrm{d}x-\\&\quad\int_0^1\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}\frac{1}{2}(\psi_l')^2\bar\psi_n''A_l^2\bar A_n\mathrm{e}^{\mathrm{i}2(\sigma_1-\sigma_2)T_1}\bar\psi_n\mathrm{d}x-\\&\quad\int_0^1\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}\bar\psi_n\psi_l'\psi_l''A_l^2\bar A_n\mathrm{e}^{\mathrm{i}2(\sigma_1-\sigma_2)T_1}\bar\psi_n\mathrm{d}x=0\end{aligned}\tag{20b}$$

将方程 (19) 代入方程 (20) 可解性条件可以写成

$$\begin{aligned}&-2\mathrm{i}\omega_l\dot A_lm_{0l}-\mathrm{i}\dot A_lg_l-2\mathrm{i}\mu\omega_lA_lm_{1l}+E_eA_l^2\bar A_lm_{2l}+\\&\quad E_eA_lA_n\bar A_nm_{3l}-bA_lA_n\bar A_nm_{4l}-\\&\quad\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}A_l^2\bar A_lm_{5l}-bA_l^2\bar A_lm_{6l}+\\&\quad\frac{a}{2}\bar A_l\mathrm{e}^{\mathrm{i}(-2\sigma_1)T_1}m_{7l}+E_eA_n^2\bar A_l\mathrm{e}^{\mathrm{i}2(\sigma_2-\sigma_1)T_1}m_{8l}-\\&\quad\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}A_n^2\bar A_l\mathrm{e}^{\mathrm{i}2(\sigma_2-\sigma_1)T_1}m_{9l}=0\end{aligned}\tag{21a}$$

$$-2\mathrm{i}\omega_n\dot{A}_n m_{0n}-\mathrm{i}\dot{A}_n g_n-2\mathrm{i}\mu\omega_n A_n m_{1n}+E_e A_n^2\bar{A}_n m_{2n}+$$
$$E_e A_n A_l\bar{A}_l m_{3n}-bA_n A_l\bar{A}_l m_{4n}-$$
$$\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}A_n^2\bar{A}_n m_{5n}-bA_n^2\bar{A}_n m_{6n}+$$
$$\frac{a}{2}\bar{A}_n\mathrm{e}^{\mathrm{i}(-2\sigma_2)T_1}m_{7n}+E_e A_l^2\bar{A}_n\mathrm{e}^{\mathrm{i}(2\sigma_1-\sigma_2)T_1}m_{8n}-$$
$$\frac{b\alpha}{(\mathrm{i}2\omega_l+\alpha)}A_l^2\bar{A}_n\mathrm{e}^{\mathrm{i}2(\sigma_1-\sigma_2)T_1}m_{9n}=0 \tag{21b}$$

式中

$$m_{0k}=\langle M\psi_k,\bar{\psi}_k\rangle,\quad g_k=-\mathrm{i}\langle G\psi_k,\bar{\psi}_k\rangle$$
$$m_{1k}=\langle\psi_k,\bar{\psi}_k\rangle$$
$$m_{2k}=\left\langle 3\psi_k'\bar{\psi}_k'\psi_k''+\frac{3}{2}(\psi_k')^2\bar{\psi}_k'',\psi_k\right\rangle$$
$$m_{3l}=\langle\psi_l'\bar{\psi}_n'\psi_n''+\psi_l'\psi_n'\bar{\psi}_n''+\psi_n'\bar{\psi}_n'\psi_l'',\bar{\psi}_l\rangle$$
$$m_{3n}=\langle\psi_n'\bar{\psi}_l'\psi_l''+\psi_n'\psi_l'\bar{\psi}_l''+\psi_l'\bar{\psi}_l'\psi_n'',\bar{\psi}_n\rangle$$
$$m_{4l}=\langle\psi_l\psi_n'\bar{\psi}_n''+\psi_l\bar{\psi}_n'\psi_n''+\bar{\psi}_n'\psi_n'\psi_l'',\bar{\psi}_l\rangle$$
$$m_{4n}=\langle\psi_n\psi_l'\bar{\psi}_l''+\psi_n\bar{\psi}_l'\psi_l''+\bar{\psi}_l'\psi_l'\psi_n'',\bar{\psi}_n\rangle$$
$$m_{5k}=\left\langle\frac{1}{2}\bar{\psi}_k''(\psi_k')^2+\bar{\psi}_k\psi_k'\psi_k'',\bar{\psi}_k\right\rangle$$
$$m_{6k}=\langle\psi_k''\psi_k'\bar{\psi}_k'+\psi_k\psi_k'\bar{\psi}_k''+\psi_k\bar{\psi}_k'\psi_k'',\bar{\psi}_k\rangle$$
$$m_{7k}=\langle\psi_k'',\bar{\psi}_k\rangle\ \ (\text{其中}\ \ k=l,n)$$
$$m_{8l}=\left\langle\frac{1}{2}\bar{\psi}_l''(\psi_n')^2+\bar{\psi}_l'\psi_n'\psi_n'',\bar{\psi}_l\right\rangle$$
$$m_{8n}=\left\langle\frac{1}{2}\bar{\psi}_n''(\psi_l')^2+\bar{\psi}_n'\psi_l'\psi_l'',\bar{\psi}_n\right\rangle$$
$$m_{9l}=\left\langle\frac{1}{2}(\psi_n')^2\bar{\psi}_l''+\bar{\psi}_l\psi_n'\psi_n'',\bar{\psi}_l\right\rangle$$
$$m_{9n}=\left\langle\frac{1}{2}(\psi_l')^2\bar{\psi}_n''+\bar{\psi}_n\psi_l'\psi_l'',\bar{\psi}_n\right\rangle \tag{22}$$

为得到直角坐标形式的平均方程，令

$$A_l(T_1)=\frac{1}{2}[x_1(T_1)-\mathrm{i}x_2(T_1)]\mathrm{e}^{\mathrm{i}\lambda_l} \tag{23a}$$
$$A_n(T_1)=\frac{1}{2}[x_3(T_1)-\mathrm{i}x_4(T_1)]\mathrm{e}^{\mathrm{i}\lambda_n} \tag{23b}$$

式中 $\lambda_l=-\sigma_1T_1-2m\pi$, $\lambda_n=-\sigma_2T_1-2m\pi$.

将方程 (23a) 和 (23b) 代入方程 (21a) 和 (21b)，分离实部和虚部，得到 1:1 内共振和主参数共振情况下直角坐标形式的平均方程

$$\dot{x}_1=-\mu x_1+(\sigma_1+\alpha)x_2+\alpha_1x_1(x_1^2+x_2^2)+$$
$$\alpha_2x_2(x_1^2+x_2^2)+\alpha_3x_1(x_3^2+x_4^2)+$$
$$\alpha_4x_2(x_3^2+x_4^2)+\alpha_5x_1(x_3^2-x_4^2)-\alpha_6x_1x_3x_4+$$
$$\alpha_6x_2(x_3^2-x_4^2)+\alpha_5x_2x_3x_4 \tag{24a}$$
$$\dot{x}_2=(-\sigma_1+\alpha)x_1-\mu x_2-\alpha_2x_1(x_1^2+x_2^2)+$$
$$\alpha_1x_2(x_1^2+x_2^2)-\alpha_4x_1(x_3^2+x_4^2)+$$
$$\alpha_3x_2(x_3^2+x_4^2)-\alpha_6x_1(x_3^2-x_4^2)+\alpha_5x_1x_3x_4-$$
$$\alpha_5x_2(x_3^2-x_4^2)+\alpha_6x_2x_3x_4 \tag{24b}$$
$$\dot{x}_3=-\mu x_3+(\sigma_2+\beta)x_4+\beta_1x_3(x_3^2+x_4^2)+$$
$$\beta_2x_4(x_3^2+x_4^2)+\beta_3x_3(x_1^2+x_2^2)+\beta_4x_4(x_1^2+x_2^2)+$$
$$\beta_5x_3(x_1^2-x_2^2)-\beta_6x_3x_1x_2+$$
$$\beta_6x_4(x_1^2-x_2^2)+\beta_5x_4x_1x_2 \tag{24c}$$
$$\dot{x}_4=(-\sigma_2+\beta)x_3-\mu x_4-\beta_2x_3(x_3^2+x_4^2)+$$
$$\beta_1x_4(x_3^2+x_4^2)-\beta_4x_3(x_1^2+x_2^2)+$$
$$\beta_3x_4(x_1^2+x_2^2)-\beta_6x_3(x_1^2-x_2^2)+\beta_5x_3x_1x_2-$$
$$\beta_5x_4(x_1^2-x_2^2)+\beta_6x_4x_1x_2 \tag{24d}$$

式中 α_i 和 β_i 为合并后的方程系数.

本节综合利用多尺度法和 Galerkin 离散法，对系统进行了摄动分析，得到了 1:1 内共振和主参数共振情形下的平均方程. 平均方程是进行数值模拟和稳定性分析的基础.

3 稳定性分析

平均方程 (24) 的定常解对应于原系统方程 (8) 的周期解，下面检验方程 (24) 的不动点附近轨道的稳定性. 令

$$\mathrm{d}x_1/\mathrm{d}t=\mathrm{d}x_2/\mathrm{d}t=\mathrm{d}x_3/\mathrm{d}t=\mathrm{d}x_4/\mathrm{d}t=0$$

得到如下方程

$$-\mu x_1+(\sigma_1+\alpha)x_2+\alpha_1x_1(x_1^2+x_2^2)+$$
$$\alpha_2x_2(x_1^2+x_2^2)+\alpha_3x_1(x_3^2+x_4^2)+$$
$$\alpha_4x_2(x_3^2+x_4^2)+\alpha_5x_1(x_3^2-x_4^2)-\alpha_6x_1x_3x_4+$$
$$\alpha_6x_2(x_3^2-x_4^2)+\alpha_5x_2x_3x_4=0 \tag{25a}$$
$$(-\sigma_1+\alpha)x_1-\mu x_2-\alpha_2x_1(x_1^2+x_2^2)+$$
$$\alpha_1x_2(x_1^2+x_2^2)-\alpha_4x_1(x_3^2+x_4^2)+$$
$$\alpha_3x_2(x_3^2+x_4^2)-\alpha_6x_1(x_3^2-x_4^2)+\alpha_5x_1x_3x_4-$$
$$\alpha_5x_2(x_3^2-x_4^2)+\alpha_6x_2x_3x_4=0 \tag{25b}$$

$$-\mu x_3+(\sigma_2+\beta)x_4+\beta_1 x_3(x_3^2+x_4^2)+ \beta_2 x_4(x_3^2+x_4^2)+\beta_3 x_3(x_1^2+x_2^2)+ \beta_4 x_4(x_1^2+x_2^2)+\beta_5 x_3(x_1^2-x_2^2)-\beta_6 x_3 x_1 x_2+ \beta_6 x_4(x_1^2-x_2^2)+\beta_5 x_4 x_1 x_2=0 \tag{25c}$$

$$(-\sigma_2+\beta)x_3-\mu x_4-\beta_2 x_3(x_3^2+x_4^2)+ \beta_1 x_4(x_3^2+x_4^2)-\beta_4 x_3(x_1^2+x_2^2)+ \beta_3 x_4(x_1^2+x_2^2)-\beta_6 x_3(x_1^2-x_2^2)+\beta_5 x_3 x_1 x_2- \beta_5 x_4(x_1^2-x_2^2)+\beta_6 x_4 x_1 x_2=0 \tag{25d}$$

为了判定系统的稳态响应的稳定性，令

$$\left.\begin{array}{ll} x_1=x_{10}+\delta_1, & x_2=x_{20}+\delta_2 \\ x_3=x_{30}+\delta_3, & x_4=x_{40}+\delta_4 \end{array}\right\} \tag{26}$$

式中 $(x_{10},\ x_{20},\ x_{30},\ x_{40})$ 是方程 (25) 的解，δ_1, δ_2,δ_3 和 δ_4 代表稳态响应的扰动，为小量.

将方程 (26) 代入方程 (24) 并根据方程 (25), 方程的线性部分为

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta_1=f_{11}\delta_1+f_{12}\delta_2+f_{13}\delta_3+f_{14}\delta_4 \tag{27a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta_2=f_{21}\delta_1+f_{22}\delta_2+f_{23}\delta_3+f_{24}\delta_4 \tag{27b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta_3=f_{31}\delta_1+f_{32}\delta_2+f_{33}\delta_3+f_{34}\delta_4 \tag{27c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta_4=f_{41}\delta_1+f_{42}\delta_2+f_{43}\delta_3+f_{44}\delta_4 \tag{27d}$$

方程的系数矩阵可以写为

$$A=\begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} \tag{28}$$

其中

$$f_{11}=-\mu+3\alpha_1 x_{10}^2+\alpha_1 x_{20}^2+2\alpha_2 x_{10}x_{20}+\alpha_3 x_{30}^2+ \alpha_3 x_{40}^2+\alpha_5(x_{30}^2-x_{40}^2)-\alpha_6 x_{30}x_{40} \tag{29a}$$

$$f_{12}=(\sigma_1+\alpha)+2\alpha_1 x_{10}x_{20}+\alpha_2 x_{10}^2+3\alpha_2 x_{20}^2+ \alpha_4(x_{30}^2+x_{40}^2)+\alpha_6(x_{30}^2-x_{40}^2)+\alpha_5 x_{30}x_{40} \tag{29b}$$

$$f_{13}=2(\alpha_3+\alpha_5)x_{10}x_{30}+2(\alpha_4+\alpha_6)x_{20}x_{30}+ \alpha_5 x_{20}x_{40}-\alpha_6 x_{10}x_{40} \tag{29c}$$

$$f_{14}=2(\alpha_3-\alpha_5)x_{10}x_{40}+2(\alpha_4-\alpha_6)x_{20}x_{40}+ \alpha_5 x_{20}x_{30}-\alpha_6 x_{10}x_{30} \tag{29d}$$

$$f_{21}=(-\sigma_1+\alpha)-3\alpha_2 x_{10}^2-\alpha_2 x_{20}^2+2\alpha_1 x_{10}x_{20}- \alpha_4(x_{30}^2+x_{40}^2)-\alpha_6(x_{30}^2-x_{40}^2)+\alpha_5 x_{30}x_{40} \tag{29e}$$

$$f_{22}=-\mu-2\alpha_2 x_{10}x_{20}+\alpha_1 x_{10}^2+3\alpha_1 x_{20}^2+\alpha_3 x_{30}^2+ \alpha_3 x_{40}^2-\alpha_5(x_{30}^2-x_{40}^2)+\alpha_6 x_{30}x_{40} \tag{29f}$$

$$f_{23}=2(\alpha_4+\alpha_6)x_{10}x_{30}+2(\alpha_3-\alpha_5)x_{20}x_{30}+ \alpha_5 x_{10}x_{40}+\alpha_6 x_{20}x_{40} \tag{29g}$$

$$f_{24}=2(\alpha_6-\alpha_4)x_{10}x_{40}+2(\alpha_3+\alpha_5)x_{20}x_{40}+ \alpha_5 x_{10}x_{30}+\alpha_6 x_{20}x_{30} \tag{29h}$$

$$f_{31}=2(\beta_3+\beta_5)x_{10}x_{30}+2(\beta_4+\beta_6)x_{10}x_{40}- \beta_6 x_{20}x_{30}+\beta_5 x_{20}x_{40} \tag{29i}$$

$$f_{32}=2(\beta_3-\beta_5)x_{20}x_{30}+2(\beta_4-\beta_6)x_{20}x_{40}- \beta_6 x_{10}x_{30}+\beta_5 x_{10}x_{40} \tag{29j}$$

$$f_{33}=-\mu+3\beta_1 x_{30}^2+\beta_1 x_{40}^2+2\beta_2 x_{30}x_{40}-\beta_6 x_{10}x_{20}+ \beta_3(x_{10}^2+x_{20}^2)+\beta_5(x_{10}^2-x_{20}^2) \tag{29k}$$

$$f_{34}=(\sigma_2+\beta)+2\beta_1 x_{30}x_{40}+\beta_2 x_{30}^2+3\beta_2 x_{40}^2+ \beta_4(x_{10}^2+x_{20}^2)+\beta_6(x_{10}^2-x_{20}^2)+\beta_5 x_{10}x_{20} \tag{29l}$$

$$f_{41}=2(\beta_4+\beta_6)x_{10}x_{30}+2(\beta_3-\beta_5)x_{10}x_{40}+ \beta_6 x_{20}x_{40}+\beta_5 x_{20}x_{30} \tag{29m}$$

$$f_{42}=2(\beta_3+\beta_5)x_{20}x_{40}+2(\beta_4-\beta_6)x_{20}x_{30}+ \beta_5 x_{10}x_{30}+\beta_6 x_{10}x_{40} \tag{29n}$$

$$f_{43}=(-\sigma_2+\beta)+2\beta_1 x_{30}x_{40}-\beta_2 x_{40}^2-3\beta_2 x_{30}^2- \beta_4(x_{10}^2+x_{20}^2)-\beta_6(x_{10}^2-x_{20}^2)+\beta_5 x_{10}x_{20} \tag{29o}$$

$$f_{44}=-\mu+3\beta_1 x_{40}^2+\beta_1 x_{30}^2-2\beta_2 x_{30}x_{40}+\beta_6 x_{10}x_{20}+ \beta_3(x_{10}^2+x_{20}^2)-\beta_5(x_{10}^2-x_{20}^2) \tag{29p}$$

令

$$p=-\mathrm{tr}(A),\quad q=\det(A) \tag{30}$$

非平凡稳态解的特征方程为

$$D(\lambda)=\lambda^2+p\lambda+q=0 \tag{31}$$

如果式 (31) 所有特征值都具有负实部，相应的稳态解是渐近稳定的；若至少有一个特征值具有正

实部，则解是不稳定的；若存在一对纯虚特征值，系统将会发生 Hopf 分岔.

4 数值模拟

数值模拟是探索黏弹性传动带系统非线性动力学特性的有效工具. 利用相图、波形图和 Poincare 截面图反映系统的非线性动力学行为，随参数变化的相图和波形图反映了参数对系统非线性动力学行为的影响. 模拟中，固定一组参数，只改变激励幅值 α，所选取的参数和初值分别为 $\mu = 0.06$, $\sigma_1 = 10.8$,

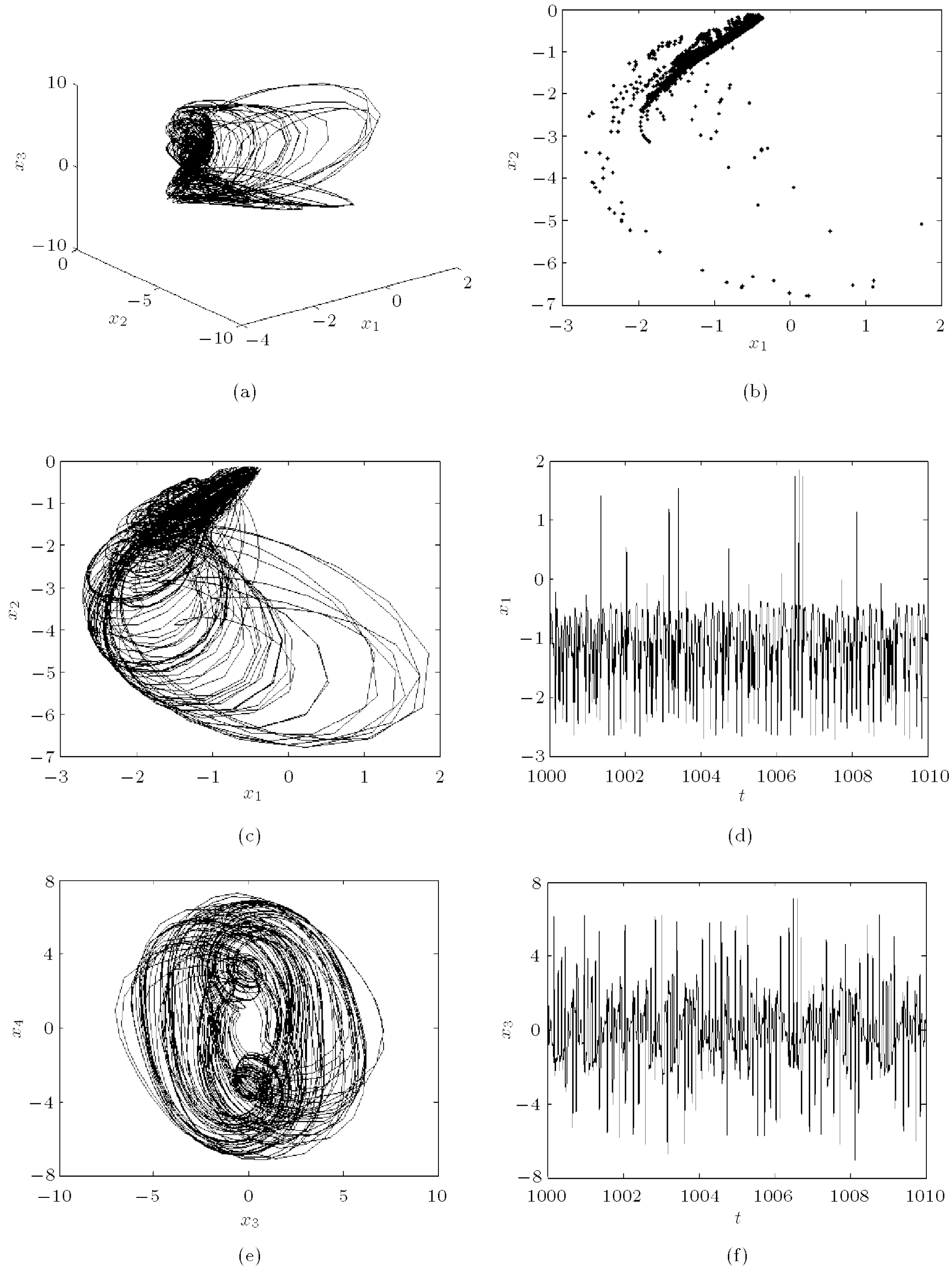

图 2 混沌运动

Fig.2 Chaotic motion

$\sigma_2 = 12.8$, $\alpha_1 = 1.1$, $\alpha_2 = 4.2$, $\alpha_3 = -3.3$, $\alpha_4 = -2.1$, $\alpha_5 = 2.47$, $\alpha_6 = 1.6$, $\beta = 25.1$, $\beta_1 = -1.1$, $\beta_2 = 2.8$, $\beta_3 = 4.3$, $\beta_4 = 10.2$, $\beta_5 = 4.08$, $\beta_6 = 2.1$, $x_{10} = -0.11$, $x_{20} = -0.01$, $x_{30} = 0.15$, $x_{40} = -0.56$. 当激励 $\alpha = 54.64$ 时系统发生混沌运动如图 2 所示，其中图 2(a) 和 2(b) 分别为 (x_1, x_2, x_3) 三维相图和 (x_1, x_2) 平面的 Poincare 截面，图 2(c) 和图 2(e) 分别表示 (x_1, x_2) 平面和 (x_3, x_4) 平面相图，图 2(d) 和 2(f) 分别表示 x_1 和 x_3 的波形图．当激励增大到 $\alpha = 58.64$ 时系统由混沌运动变为概周期运动如图 3 所示．持

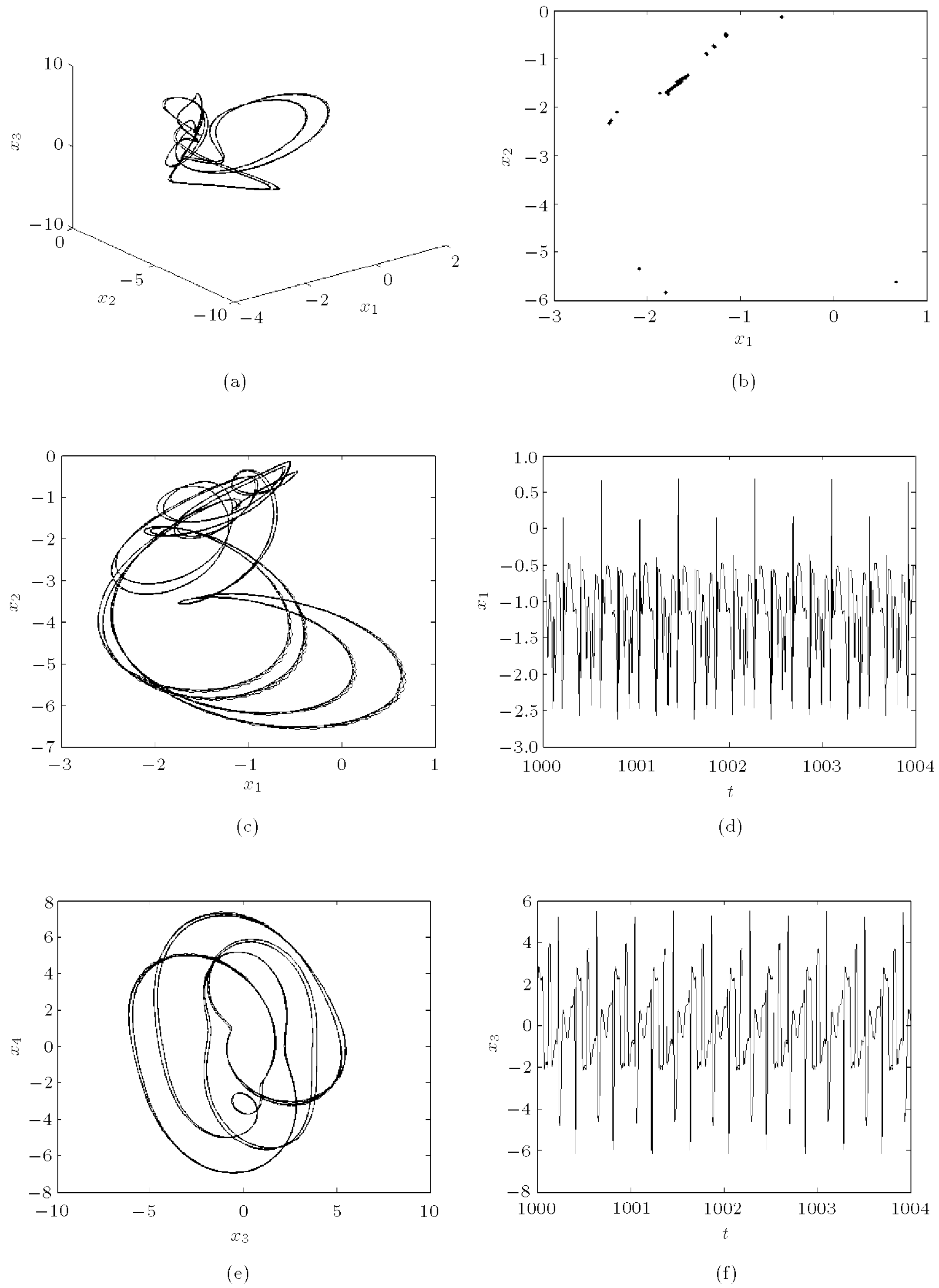

图 3 概周期运动

Fig.3 Quasi-periodic motion

续增大激励幅值到 $\alpha = 58.74$ 和 $\alpha = 59.64$, 系统依次发生二倍周期和周期运动，如图 4 和图 5 所示.

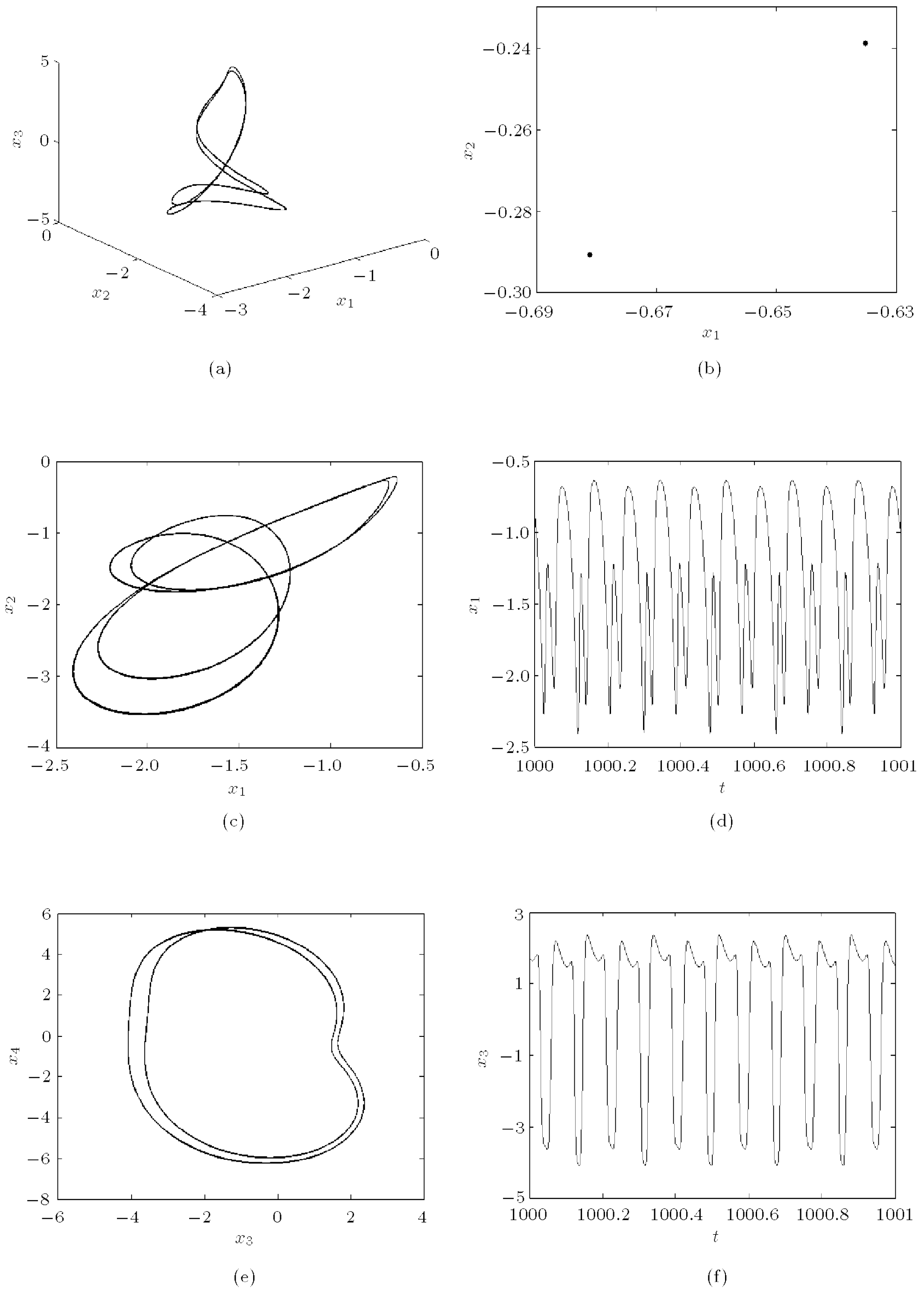

图 4 两倍周期运动

Fig.4 Periodic-2 motion

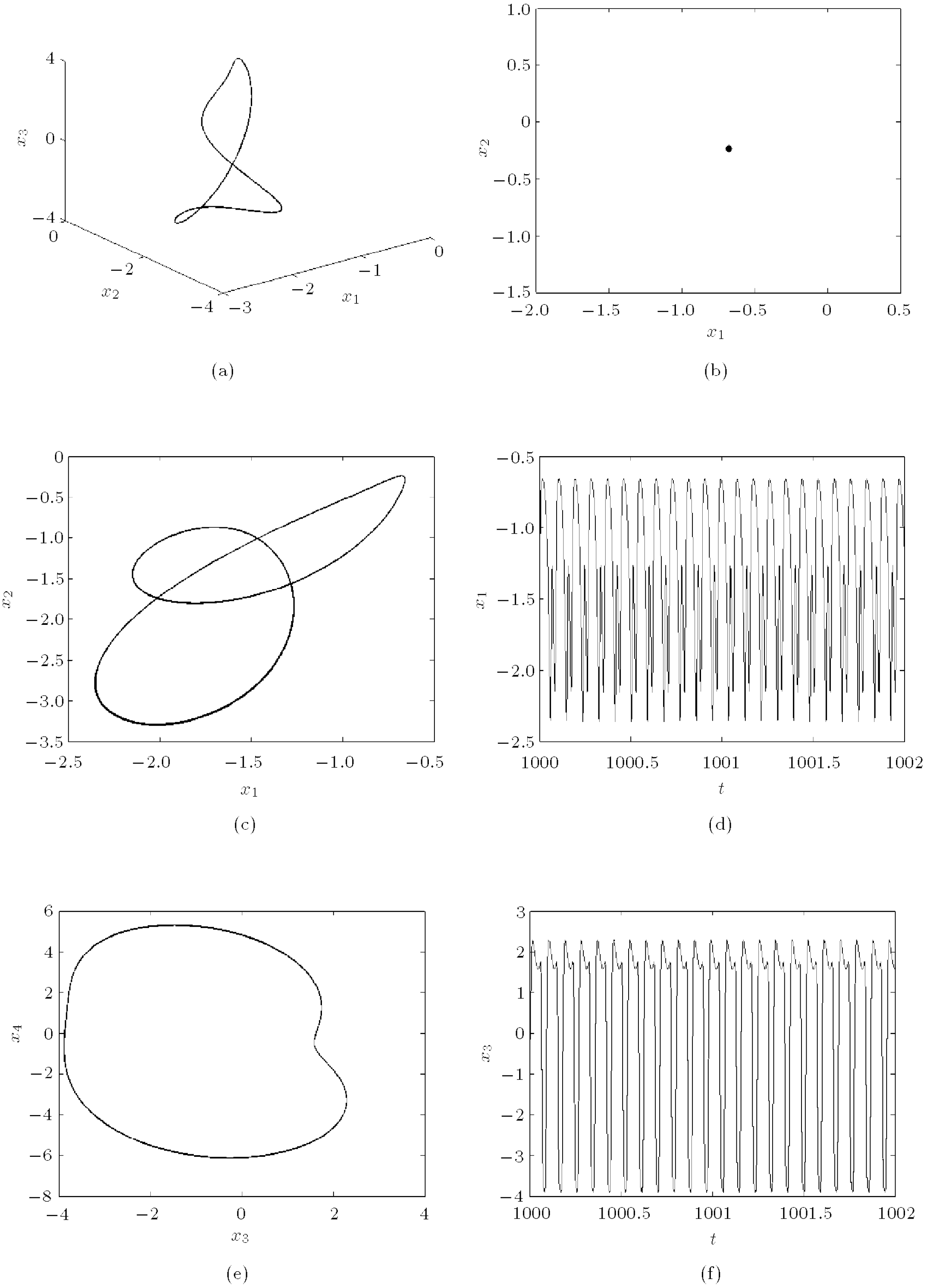

图 5 周期运动

Fig.5 Periodic motion

5 结 论

本文首次研究了黏弹性传动带在 1:1 内共振时非平面非线性横向振动特性. 首先, 应用 Hamilton 建立了具有线性外阻尼情况下的黏弹性传动带非平面横向振动的动力学方程, 黏弹性材料采用积分本构模型描述. 然后, 综合利用多尺度法和 Galerkin 离散法进行摄动分析, 并得到黏弹性传动带系统在 1:1

内共振和主参数共振时直角坐标形式的平均方程.

基于平均方程的定常解对应于原始系统的周期解，通过讨论平均方程的定常解的稳定性来反映原始系统解的稳定性. 最后，对平均方程进行数值模拟，揭示了黏弹性传动带系统的非线性振动行为. 数值模拟结果发现黏弹性传动带系统存在混沌、概周期、两倍周期运动和周期运动. 随着激励幅值 α 的变化，系统的动力学响应呈现出一定的规律，即混沌运动 $\to$ 概周期运动 $\to$ 两倍周期运动 $\to$ 周期运动. 由此可见，非线性因素导致黏弹性传动带的动力学行为更为复杂，呈现出混沌现象. 通过改变激励幅值 α 可以控制传动带系统的非线性动力学行为.

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TRANSVERSE NONLINEAR NONPLANAR DYNAMICS OF AN AXIALLY MOVING VISCOELASTIC BELT WITH INTEGRAL CONSTITUTIVE LAW [1)]

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Abstract In this paper, the problem of the transverse nonlinear nonplanar oscillations of an axially moving viscoelastic belt with the integral constitutive law are investigated in the case of 1:1 internal resonance. The governing equations of this problem are firstly derived with the generalized Hamilton's principle to obtain the in-plane and out-of-plane transverse nonlinear oscillations of the axially moving viscoelastic belt neglecting the axially deformation. Perturbation analyses are carried out on these partial differential governing equations with the multiscale method and the Galerkin's approach to obtain four-dimensional averaged equations and to analyze the stabilities of the solution in the dynamic system. The simulation results show the periodic motion, the quasi-periodic motion and the chaotic motion in the transverse nonlinear nonplanar oscillations of the axially moving viscoelastic belt.

Key words viscoelastic belt, nonlinear, transverse oscillation, internal resonance, chaos

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