磁电热弹耦合材料三维多裂纹超奇异积分法

朱伯靖 *,† 秦太验 *,1)

*(中国农业大学理学院，北京 100083) †(中国科学院研究生院地球科学学院，北京 100049)

摘要 用超奇异积分方程法将多场耦合载荷作用下磁电热弹耦合材料内含任意形状和位置三维多裂纹问题转化为求解一以广义位移间断为未知函数的超奇异积分方程组问题，退化得到内含任意形状平行三维多裂纹问题的超奇异积分方程组；推导出平行三维多裂纹问题的裂纹前沿广义奇异应力场解析表达式、定义了广义 (应力、应变能) 强度因子和广义能量释放率；应用有限部积分概念及体积力法，为超奇异积分方程组建立了数值求解方法，编制了 FORTRAN 程序，以平行双裂纹为例，通过典型算例，研究了广义 (应力、应变能) 强度因子随裂纹位置、裂纹形状及材料参数变化规律，得到裂纹断裂评定准则．最后，分析了裂纹间干扰、屏蔽作用及其在工程实际中的应用.

关键词 三维多裂纹，磁电热弹耦合材料，超奇异积分，应力强度因子

中图分类号： O346.1, O302 **文献标识码：** A **文章编号：** 0459-1879(2008)01-0046-13

引 言

从 Brewster, Curie 兄弟， Vaslek 到 Davis, Vandenboomagaard, Tinkham, Aboudi, 磁电热弹耦合问题一直受到广泛关注 [1~6]; Aboudi, Gao 等 [7~11] 对磁电热弹耦合材料进行了基础性理论研究； Zhen 和 Zhou 建立了电磁材料结构非线性力学行为的完整理论框架，预测了铁磁材料典型实验现象磁力表达式 [12,13]，给出了多场耦合非线性本构关系 [14,15]，对电磁材料结构行为给出定量预测 [16,17]; Zhou 等对磁电热弹耦合作用下磁通跳跃现象给出成功预测 [18,19]; Sih[20~23] 最早提出应变能强度因子理论，并把它应用到复合材料断裂力学研究中，做了许多开创性工作 [24]; 杨卫 [25,26], Li[27], 刘又文 [28] 等学者也从不同方面对磁电热弹耦合材料研究做出了突出贡献．以上这些成果为磁电热弹耦合材料断裂问题研究奠定了良好的理论基础．但鉴于数值方法、理论模型、数学力学和物理学上的困难，特别是数值方法 (计算量与精度) 、理论模型 (三维裂纹扩展模型) 、数学 (公式庞杂) 及力学和物理学 (问题复杂) 上的困难，两相或多相耦合场作用方面研究虽然取得一些成果 [20,29~37]，但三维多裂纹干扰及相互作用问题的研究较少.

本文根据广义位移基本解 [38] ，将磁电热弹耦合材料内含任意形状和位置三维多裂纹问题转化为求解一以广义位移间断为未知函数的超奇异积分方程组问题，退化得到内含任意形状平行三维多裂纹问题的超奇异积分方程组；应用主部分析法，通过分析平行多裂纹问题的裂纹前沿广义应力奇性指数，得到了广义奇性应力场解析表达式，定义了广义 (应力、应变能) 强度因子和能量释放率；应用有限部积分概念和体积力法，对超奇异积分方程组建立了数值方法，编写了广义 (应力、应变能) 强度因子计算专用 FORTRAN 程序；以典型平行三维双裂纹为例，研究了广义 (应力、应变能) 强度因子与裂纹形状、裂纹间位置关系，得到裂纹断裂评定准则，分析了多裂纹间干扰和屏蔽作用及其在工程实际中的应用.

1 数学模型及超奇异积分方程组

如图 1 所示，设磁电热弹耦合材料空间区域 Ω 中含有任意形状和位置的三维多裂纹. 选择整体坐标系为空间直角坐标系 $x_i(i=1,2,3)$, 在裂纹面 $S_{\hat{i}}(\hat{i}=1,2,\cdots,n)$ 表面上分别作用力载荷 $p_j^{\hat{i}}(j=1,2,3)$, 电载荷 $q^{\hat{i}}$, 磁载荷 $b^{\hat{i}}$ 和热载荷 $\rho^{\hat{i}}$. 为了研究问题方便，建立局部坐标系 $x_i^{\hat{i}}$, 裂纹 $\hat{i}$ 位于局部系坐标轴 $x_1^{\hat{i}}x_2^{\hat{i}}$ 构成的平面内，同时与局部系坐标轴 $x_3^{\hat{i}}$ 垂直，裂纹中心点在局部系中坐标为 $O^{\hat{i}}(\xi_i^{\hat{i}})$, 局部系坐标轴 $x_i^{\hat{i}}$

2007–06–13 收到第 1 稿， 2007–09–24 收到修改稿.
1) E-mail: tyqin@cau.edu.cn

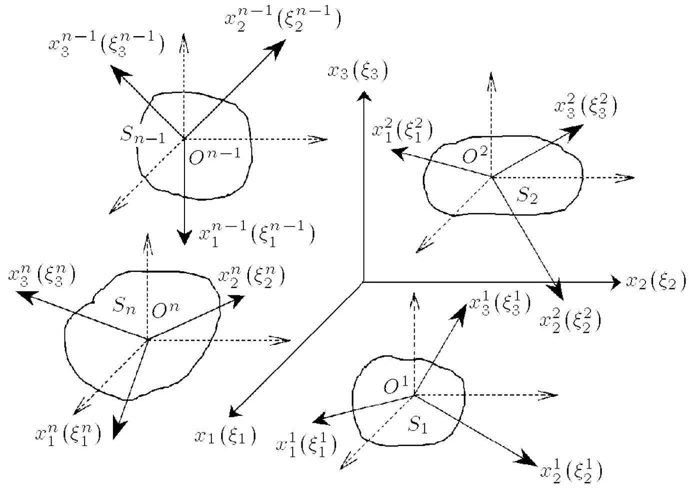


图 1　磁电热弹耦合材料内含任意形状和位置三维多裂纹示意图

Fig.1　Multiple interacting dislocations in electromagnetothermoelastic multiphase composites

与整体系坐标轴 x_i 夹角 $\theta_i^{\hat{i}}$ $(x_i, x_i^{\hat{i}})$. 利用 Somigliana 公式, 磁电热弹耦合材料中任意一点 $p(x_1,x_2,x_3)$ 处广义位移 $U_I(p)$ 可表示为

$$U_I=\int_{\Omega}U_{IJ}^0f_J\mathrm{d}V-\int_{\Gamma}(T_{IJ}^0U_J^0+U_{IJ}^0T_J^0)\mathrm{d}s+\underbrace{\left(\int_{S_1^++S_1^-}(U_{IJ}^1T_J^1-T_{IJ}^1U_J^1)\mathrm{d}s+\int_{S_2^++S_2^-}(U_{IJ}^2T_J^2-T_{IJ}^2U_J^2)\mathrm{d}s+\cdots+\int_{S_{n-1}^++S_{n-1}^-}(U_{IJ}^{n-1}T_J^{n-1}-T_{IJ}^{n-1}U_J^{n-1})\mathrm{d}s+\int_{S_n^++S_n^-}(U_{IJ}^nT_J^n-T_{IJ}^nU_J^n)\mathrm{d}s\right)}_{n} \tag{1}$$

其中广义面力 $T_J^{\tilde{i}}(q)$ $(\tilde{i}=0,1,2,\cdots,n)$ 具体表达式为

$$T_J^{\tilde{i}}=\begin{cases}p_j^{\tilde{i}}=\sigma_{jl}^{\tilde{i}}n_l\,, & J=j=1,2,3\\ q^{\tilde{i}}=D_l^{\tilde{i}}n_l\,, & J=4\\ b^{\tilde{i}}=B_l^{\tilde{i}}n_l\,, & J=5\\ \rho^{\tilde{i}}=\vartheta_{,J}^{\tilde{i}}n_i\,, & J=6\end{cases} \tag{2}$$

广义位移和面力基本解 $U_{IJ}^{\tilde{i}}$ 和 $T_{IJ}^{\tilde{i}}$ 为已知函数, 两者间关系为 $T_{IJ}^{\tilde{i}}=E_{kJMn}U_{IM,n}^{\tilde{i}}n_k$. 引入裂纹面 $S_{\hat{i}}$ 上广义位移间断 $\tilde{U}_J^{\hat{i}}$, 应用边界条件 (外边界 Γ 和裂纹面 $S_{\hat{i}}$ 处边界), 让源点 p 不断向边界趋近, 当达到边界时用 P 点来代替, 从式 (1) 可得到以下边界积分方程

$$C_{IJ}U_I=-\underbrace{\left(\int_{S_1^+}T_{IJ}^{1+}\tilde{U}_J^1\mathrm{d}s+\int_{S_2^+}T_{IJ}^{2+}\tilde{U}_J^2\mathrm{d}s+\int_{S_3^+}T_{IJ}^{3+}\tilde{U}_J^3\mathrm{d}s+\cdots+\int_{S_{n-1}^+}T_{IJ}^{(n-1)+}\tilde{U}_J^{n-1}\mathrm{d}s+\int_{S_n^+}T_{IJ}^{n+}\tilde{U}_J^n\mathrm{d}s\right)}_{n}+\int_{\Gamma}(U_{IJ}^0T_J^0-T_{IJ}^0U_J^0)\mathrm{d}s+\int_{\Omega}U_{IJ}^0b_J\mathrm{d}V \tag{3}$$

广义应力张量 $\sum_{iJ}$ 可表示为

$$\sum_{iJ}=-\underbrace{\left(\int_{S_1^+}S_{KiJ}^{1+}\tilde{U}_K^1\mathrm{d}s+\int_{S_2^+}S_{KiJ}^{2+}\tilde{U}_K^2\mathrm{d}s+\int_{S_3^+}S_{KiJ}^{3+}\tilde{U}_K^3\mathrm{d}s+\cdots+\int_{S_{n-1}^+}S_{KiJ}^{n-1+}\tilde{U}_K^{n-1}\mathrm{d}s+\int_{S_n^+}S_{KiJ}^{n+}\tilde{U}_K^n\mathrm{d}s\right)}_{n}+\int_{\Gamma}(D_{KiJ}^0T_K^0-S_{KiJ}^0U_K^0)\mathrm{d}s+\int_{\Omega}D_{KiJ}^0f_K\mathrm{d}V \tag{4}$$

其中 $S_{KiJ}^{\hat{i}}=-E_{iJMn}T_{MK,n}^{\hat{i}}$, $D_{KiJ}^{\hat{i}}=-E_{iJMn}U_{MK,n}^{\hat{i}}$; 常数 C_{IJ} 与边界点 P 相关, 应用裂纹面的广义边界条件, 可以得到以下超奇异积分方程组.

$$\underbrace{\left(\fint_{S_1^+} S_{KiJ}^{1+}\tilde{U}_K^1 \mathrm{d}s + \fint_{S_2^+} S_{KiJ}^{2+}\tilde{U}_K^2 \mathrm{d}s + \fint_{S_3^+} S_{KiJ}^{3+}\tilde{U}_K^3 \mathrm{d}s + \cdots + \fint_{S_{n-1}^+} S_{KiJ}^{(n-1)+}\tilde{U}_K^{n-1} \mathrm{d}s + \fint_{S_n^+} S_{KiJ}^{n+}\tilde{U}_K^n \mathrm{d}s\right)}_{n} =$$

$$\int_{\Gamma}(D_{KiJ}^0 T_K^0 - S_{KiJ}^0 U_K^0)\mathrm{d}s + \int_{\Omega} D_{KiJ}^0 f_K \mathrm{d}s \tag{5}$$

其中 $\fint$ 为超奇异积分符号，必须通过有限部积分才可以求解．对于空间无限体内含裂纹问题，设 Q 点为空间 Ω 任意一点，该点在整体系下沿坐标轴方向广义位移可表示为

$$U_K(Q) = -\int_{S_{\hat{i}}^+} T_{KI}(P_{\hat{i}}, Q)\tilde{U}_I(P_{\hat{i}})\mathrm{d}S_{\hat{i}} = -\int_{S_{\hat{i}}^+} T_{KI}^{\hat{i}}(P_{\hat{i}}, Q)\tilde{U}_I^{\hat{i}}(P_{\hat{i}})\mathrm{d}S_{\hat{i}} \tag{6}$$

裂纹 I 处整体系和局部系下广义位移间断 $\tilde{U}_I$ 和 $\tilde{U}_I^{\hat{i}}$ 具体表达式为

$$\tilde{U}_I(P_{\hat{i}}) = \begin{cases} \tilde{u}_i(P_{\hat{i}}) = u_i^+(P_{\hat{i}}) - u_i^-(P_{\hat{i}}), & I = i = 1,2,3 \\ \tilde{\phi}_I(P_{\hat{i}}) = \phi_I^+(P_{\hat{i}}) - \phi_I^-(P_{\hat{i}}), & I = 4 \\ \tilde{\varphi}_I(P_{\hat{i}}) = \varphi_I^+(P_{\hat{i}}) - \varphi_I^-(P_{\hat{i}}), & I = 5 \\ \tilde{\vartheta}_I(P_{\hat{i}}) = \tilde{\vartheta}_I^+(P_{\hat{i}}) - \tilde{\vartheta}_I^-(P_{\hat{i}}), & I = 6 \end{cases}$$

$$\tilde{U}_I^{\hat{i}}(P_{\hat{i}}) = \begin{cases} \tilde{u}_i^{\hat{i}}(P_{\hat{i}}) = u_i^{\hat{i}+}(P_{\hat{i}}) - u_i^{\hat{i}-}(P_{\hat{i}}), & I = i = 1,2,3 \\ \tilde{\phi}_I^{\hat{i}}(P_{\hat{i}}) = \phi_I^{\hat{i}+}(P_{\hat{i}}) - \phi_I^{\hat{i}-}(P_{\hat{i}}), & I = 4 \\ \tilde{\varphi}_I^{\hat{i}}(P_{\hat{i}}) = \varphi_I^{\hat{i}+}(P_{\hat{i}}) - \varphi_I^{\hat{i}-}(P_{\hat{i}}), & I = 5 \\ \tilde{\vartheta}_I^{\hat{i}}(P_{\hat{i}}) = \tilde{\vartheta}_I^{\hat{i}+}(P_{\hat{i}}) - \tilde{\vartheta}_I^{\hat{i}-}(P_{\hat{i}}), & I = 6 \end{cases}$$

整体系和局部系下广义面力基本解 T_{KI} 和 $T_{KI}^{\hat{i}}$．它们与广义位移基本解之间的关系为

$$T_{KI}(P_{\hat{i}}, Q) = E_{IIMn}U_{KM,n}(P_{\hat{i}}, Q)n_I$$

$$T_{KI}^{\hat{i}}(P_{\hat{i}}, Q) = E_{IIMn}U_{KM,n}^{\hat{i}}(P_{\hat{i}}, Q)n_I = \boldsymbol{H}_{KT}^{\hat{i}}\boldsymbol{H}_{ILIL}^{\hat{i}}T_{TL}(P_{\hat{i}}, Q) = \boldsymbol{H}_{KT}^{\hat{i}}\boldsymbol{H}_{IL}^{\hat{i}}E_{IILn}U_{KL,n}(P_{\hat{i}}, Q)n_I$$

$U_{\hat{K}M}$ 表达式可参考文献 [41]，坐标旋转变换张量 $\boldsymbol{H}_{KT}^{\hat{i}\hat{j}}, \boldsymbol{H}_{KT}^{\hat{i}}$ 及坐标平动变换张量 $x_i^{\hat{i}\hat{j}}, x_i^{\hat{i}}$ 定义如下

$$\boldsymbol{H}_{KT}^{\hat{i}\hat{j}} = \begin{cases} \iota_{KT}^{\hat{i}\hat{j}}, & K,T=1,2,3 \\ 1, & K,T=4,5,6 \end{cases}, \quad x_i^{\hat{i}\hat{j}} = \boldsymbol{H}_{KT}^{\hat{l}\hat{j}}(x_i^{\hat{l}} - O_i^{\hat{i}\hat{j}})$$

$$\boldsymbol{H}_{KT}^{\hat{i}} = \begin{cases} \iota_{KT}^{\hat{i}}, & K,T=1,2,3 \\ 1, & K,T=4,5,6 \end{cases}, \quad x_i^{\hat{i}} = \boldsymbol{H}_{KT}^{\hat{l}\hat{j}}(x_i^{\hat{l}} - O_i)$$

其中 $\iota_{KT}^{\hat{i}\hat{j}} = \cos(x_K^{\hat{i}}, x_T^{\hat{j}})$, $\iota_{KT}^{\hat{i}} = \cos(x_K^{\hat{i}}, x_T)$．利用位移 - 应变关系和 Hooke 定律，可得到 Q 点以广义基本解表示的广义应力分量

$$\sum_{IJ}(Q) = -\int_{S_{\hat{i}}^+} S_{KIJ}^{\hat{i}}(P_{\hat{i}}, Q)\tilde{U}_K^{\hat{i}}(Q)\mathrm{d}S_{\hat{i}}^+ \tag{7}$$

利用主部分析法，经过繁杂数学推导，可得到无限体中内含任意形状和位置三维多裂纹问题的超奇异积分方程组．该方程组由 n 组超奇异积分方程组共同构成 (共 $n \times 6$ 个独立方程)，为简化起见，本文只给出第 $\hat{i}$ 组超奇异积分方程，其具体表达式如下

$$\hat{i}\left[\begin{array}{l} \fint_{S_{\hat{i}}^+}\Big(r^{-3}(c_{44}^2 D_0 s_0^2(\delta_{\bar{\alpha}\bar{\beta}} - 3r_{,\bar{\alpha}}r_{,\bar{\beta}}) + (\delta_{\alpha\beta} - 3r_{,\alpha}r_{,\beta})\sum_{i=1}^{5}\rho_i^2 t_i^2\Big)\tilde{u}_\beta^{\hat{i}} + 3r^{-4}r_{,\alpha}\sum_{i=1}^{5}\lambda_{33}s_i^2 t_i^1\tilde{u}_6^{\hat{i}}\Big)\mathrm{d}s + \\ \quad \int_{S_{\hat{i}}^+} K_{\alpha i}^{\hat{i}}\tilde{u}_i^{\hat{i}}\mathrm{d}s + \sum_{\hat{i}=1}^{\hat{n}-2}\int_{S_{\hat{i}}^+} K_{\alpha i}^{\hat{i}}\tilde{u}_i^{\hat{i}}\mathrm{d}s = -p_\alpha^{\hat{i}} \\ \fint_{S_{\hat{i}}^+}\Big(r^{-2}r_{,\alpha}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1\tilde{u}_\alpha^{\hat{i}} + r^{-3}\sum_{n=3}^{5}\sum_{i=1}^{5}\rho_i^m t_i^t\tilde{u}_n^{\hat{i}} + 3r^{-4}\lambda_{3\alpha}r_{,\alpha}\sum_{i=1}^{5}s_i^2\lambda_i^{\vartheta}\rho_i^m\tilde{u}_6^{\hat{i}}\Big)\mathrm{d}s + \\ \quad \int_{S_{\hat{i}}^+} K_{mi}^{\hat{i}}\tilde{u}_i^{\hat{i}}\mathrm{d}s + \sum_{\hat{i}=1}^{\hat{n}-2}\int_{S_{\hat{i}}^+} K_{mi}^{\hat{i}}\tilde{u}_i^{\hat{i}}\mathrm{d}s = -p_m^{\hat{i}} \\ \fint_{S_{\hat{i}}^+}\Big(r^{-2}(\delta_{\alpha\beta} - 3r_{,\alpha}r_{,\beta})\sum_{i=1}^{5}A_i^{\Upsilon}\lambda_{3\beta}t_i^2\tilde{u}_\beta^{\hat{i}} + 3r^{-4}\lambda_{3\alpha}r_{,\alpha}\sum_{i=1}^{5}A_i^{\Upsilon}s_i^2\lambda_i^{\vartheta}\lambda_{33}\rho_i^6\tilde{u}_6^{\hat{i}}\Big)\mathrm{d}s + \\ \quad \int_{S_{\hat{i}}^+} K_{6i}^{\hat{i}}\tilde{u}_i^{\hat{i}}\mathrm{d}s + \sum_{\hat{i}=1}^{\hat{n}-2}\int_{S_{\hat{i}}^+} K_{6i}^{\hat{i}}\tilde{u}_i^{\hat{i}}\mathrm{d}s = -p_6^{\hat{i}} \end{array}\right. \tag{8}$$

超奇异方程组完全表达式读者可根据本文方法自行得出．核函数 $K_{ij}^{\hat{i}}$ 及其它参数表达式见附录．如果 $\theta_i^{\hat{i}}(x_i, x_i^{\hat{i}}) = 0$ 则以上超奇异积分方程组就退化为内含任意形状平行三维多裂纹问题情形．对于更特殊情况 ($\theta_i^{\hat{i}}(x_i, x_i^{\hat{i}}) = 0$, $n = 2$) 可得到两平行三维裂纹问题超奇异积分方程组具体表示式

$$\left.\begin{aligned}
&\fint_{S_1^+}\Big(r^{-3}\Big(c_{44}^2 D_0 s_0^2(\delta_{\bar\alpha\bar\beta} - 3r_{,\bar\alpha} r_{,\bar\beta}) + (\delta_{\alpha\beta} - 3r_{,\alpha} r_{,\beta})\sum_{i=1}^5 \rho_i^2 t_i^2\Big)\tilde{u}_\beta^1 + 3r^{-4} r_{,\alpha}\sum_{i=1}^5 \lambda_{33} s_i^2 t_i^1 \tilde{u}_6^1\Big)\mathrm{d}s + \\
&\qquad \int_{S_2^+} K_{\alpha i}^1 \tilde{u}_i^2 \mathrm{d}s = -p_\alpha^1 \\
&\fint_{S_1^+}\Big(r^{-2} r_{,\alpha}\sum_{i=1}^5 A_i^{\Upsilon} t_i^2 \rho_i^1 \tilde{u}_\alpha^1 + r^{-3}\sum_{n=3}^5\sum_{i=1}^5 \rho_i^m t_i^t \tilde{u}_n^1 + 3r^{-4}\lambda_{3\alpha} r_{,\alpha}\sum_{i=1}^5 s_i^2 \lambda_i^{\vartheta} \rho_i^m \tilde{u}_6^1\Big)\mathrm{d}s + \int_{S_2^+} K_{mi}^1 \tilde{u}_i^2 \mathrm{d}s = -p_m^1 \\
&\fint_{S_1^+}\Big(r^{-2}(\delta_{\alpha\beta} - 3r_{,\alpha} r_{,\beta})\sum_{i=1}^5 A_i^{\Upsilon}\lambda_{3\beta} t_i^2 \tilde{u}_\beta^1 + 3r^{-4}\lambda_{3\alpha} r_{,\alpha}\sum_{i=1}^5 A_i^{\Upsilon} s_i^2 \lambda_i^{\vartheta}\lambda_{33}\rho_i^6 \tilde{u}_6^1\Big)\mathrm{d}s + \int_{S_2^+} K_{6i}^1 \tilde{u}_i^2 \mathrm{d}s = -p_6^1 \\
&\fint_{S_2^+}\Big(r^{-3}\Big(c_{44}^2 D_0 s_0^2(\delta_{\bar\alpha\bar\beta} - 3r_{,\bar\alpha} r_{,\bar\beta}) + (\delta_{\alpha\beta} - 3r_{,\alpha} r_{,\beta})\sum_{i=1}^5 \rho_i^2 t_i^2\Big)\tilde{u}_\beta^2 + 3r^{-4} r_{,\alpha}\sum_{i=1}^5 \lambda_{33} s_i^2 t_i^1 \tilde{u}_6^2\Big)\mathrm{d}s + \\
&\qquad \int_{S_1^+} K_{\alpha i}^2 \tilde{u}_i^1 \mathrm{d}s = -p_\alpha^2 \\
&\fint_{S_2^+}\Big(r^{-2} r_{,\alpha}\sum_{i=1}^5 A_i^{\Upsilon} t_i^2 \rho_i^1 \tilde{u}_\alpha^1 + r^{-3}\sum_{n=3}^5\sum_{i=1}^5 \rho_i^m t_i^t \tilde{u}_n^2 + 3r^{-4}\lambda_{3\alpha} r_{,\alpha}\sum_{i=1}^5 s_i^2 \lambda_i^{\vartheta} \rho_i^m \tilde{u}_6^2\Big)\mathrm{d}s + \int_{S_1^+} K_{mi}^2 \tilde{u}_i^1 \mathrm{d}s = -p_m^2 \\
&\fint_{S_2^+}\Big(r^{-2}(\delta_{\alpha\beta} - 3r_{,\alpha} r_{,\beta})\sum_{i=1}^5 A_i^{\Upsilon}\lambda_{3\beta} t_i^2 \tilde{u}_\beta^2 + 3r^{-4}\lambda_{3\alpha} r_{,\alpha}\sum_{i=1}^5 A_i^{\Upsilon} s_i^2 \lambda_i^{\vartheta}\lambda_{33}\rho_i^6 \tilde{u}_6^2\Big)\mathrm{d}s + \int_{S_1^+} K_{6i}^2 \tilde{u}_i^1 \mathrm{d}s = -p_6^2
\end{aligned}\right\} \quad (9)$$

核函数 K_{mn}^{α} 具体表达式见附录.

2 广义应力强度因子、奇性应力场、应变能强度因子和能量释放率

应用文献 [39] 中方法，广义应力强度因子可定义为

$$\left.\begin{aligned}
K_1^{\hat{i}} &= \lim_{r\to 0}\sqrt{2r}\,\sigma_{33}^{\hat{i}}(r,\theta)\big|_{\theta=0} \\
K_2^{\hat{i}} &= \lim_{r\to 0}\sqrt{2r}\,\sigma_{32}^{\hat{i}}(r,\theta)\big|_{\theta=0} \\
K_3^{\hat{i}} &= \lim_{r\to 0}\sqrt{2r}\,\sigma_{31}^{\hat{i}}(r,\theta)\big|_{\theta=0}
\end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned}
K_4^{\hat{i}} &= \lim_{r\to 0}\sqrt{2r}\,D_3^{\hat{i}}(r,\theta)\big|_{\theta=0} \\
K_5^{\hat{i}} &= \lim_{r\to 0}\sqrt{2r}\,B_3^{\hat{i}}(r,\theta)\big|_{\theta=0} \\
K_6^{\hat{i}} &= \lim_{r\to 0}\sqrt{2r}\,\vartheta_3^{\hat{i}}(r,\theta)\big|_{\theta=0}
\end{aligned}\right\} \quad (11)$$

应用主部分析法，经过繁杂的数学推导，裂纹前沿广义奇性应力场可表示为

$$\sigma_{13}^{\hat{i}} = c_{44}^2 D_0 s_0^2 \frac{\pi g_1^{\hat{i}}}{\sqrt{r r_0}}\cos\frac{\theta_0}{2} \quad (12)$$

$$\begin{aligned}
\sigma_{23}^{\hat{i}} = \sum_{i=1}^5 \frac{\pi}{\sqrt{r r_i}}\Big\{ g_6^{\hat{i}}\lambda_{33}\Big[15\lambda_{33} s_i^2 \cot\theta_i\Big(3\cos\frac{\theta_i}{4} - {} \\
3\sin^2\frac{\theta_i}{2}\cos\frac{\theta_i}{2} - \frac{1}{4}\sin^2\theta_i\cos\frac{3\theta_i}{2}\Big) - \frac{2\lambda_{33}}{r^2 r_i^2}\sin^{-1}\frac{\theta_i}{2} + {} \\
\frac{\lambda_{32}}{2 r r_i \sin\theta}\Big(2\cos^{-1}\frac{\theta_i}{2} - 3\cos\frac{\theta_i}{2} - \cos\frac{5\theta_i}{2}\Big)\Big] + {} \\
\rho_i^2 \pi\cos\frac{\theta_i}{2}\Big(t_2 g_2 + 4\sum_{m=3}^5 t_i^m g_m^{\hat{i}}\Big)\Big\}
\end{aligned} \quad (13)$$

$$\begin{aligned}
\dot{\sigma}_{3n}^{\hat{i}} = \sum_{i=1}^5 \rho_i^n s_i \pi\Big[\frac{1}{\sqrt{r r_i}}\Big(t_i^2 g_2^{\hat{i}}\cot\theta_i\cos^{-1}\frac{\theta_i}{2} + {} \\
6\cot\frac{\theta_i}{2}\sum_{m=3}^5 t_i^m g_m^{\hat{i}}\Big) + g_6^{\hat{i}} s_i \lambda_i^{\vartheta}\sqrt{r r_i}\cot\theta_i\cos\frac{\theta_i}{2}\Big)\Big] + {} \\
2\pi A_i^{\Upsilon}\iota_{33}\Big[\sqrt{r r_i}\cos\frac{\theta_i}{2}\Big(g_2^{\hat{i}}\cot\theta_i + \sum_{m=3}^5 t_i^m g_m^{\hat{i}}\Big) + {} \\
\frac{s_i^2\lambda_{33}\lambda_i^{\vartheta} g_6^{\hat{i}}}{\sqrt{r r_i}}\cos\frac{\theta_i}{2}\Big],\quad n = 3\sim 5
\end{aligned} \quad (14)$$

$$\begin{aligned}
\vartheta_3^{\hat{i}} = \sum_{i=1}^5 \frac{A_i^{\Upsilon}\pi}{\sqrt{r r_i}}\Big[\cos\frac{\theta_i}{2}(\lambda_{32} + 4\lambda_{33} s_i^2)\sum_{m=2}^5 t_i^m g_m^{\hat{i}} + {} \\
\frac{4\lambda_{32} t_i^2 g_2^{\hat{i}} s_i^2 \lambda_i^{\vartheta}}{3}\Big(2\lambda_{32}\cos\frac{\theta_i}{2} + \frac{3\lambda_{33}}{r^2 r_i^2}\sin^{-1}\frac{\theta_i}{2}\Big).
\end{aligned}$$

$$15rr_ig_6^{\hat{i}}\lambda_{32}s_i^2\lambda_i^{\vartheta}\cot\theta_i\Big(3\cos\frac{\theta_i}{4}-6\cos\frac{\theta_i}{2}\sin^2\frac{\theta_i}{2}-\frac{1}{4}\sin^2\theta_i\cos\frac{3\theta_i}{2}\Big)\Big(2s_i^2\lambda_{33}+\frac{\lambda_{32}}{rr_i\sin\theta_i}\Big)\Big] \quad (15)$$

广义能量释放率可表示为

$$G^{\hat{i}}=\pi(G_1^{\sigma\hat{i}}+G_2^{\sigma\hat{i}}+G_3^{\sigma\hat{i}}+G_4^{D\hat{i}}+G_5^{B\hat{i}}+G_6^{\Upsilon\hat{i}})/2 \quad (16)$$

其中

$$G_1^{\sigma\hat{i}}=\frac{K_1^{\hat{i}}[K_4^{*\hat{i}}(K_{34}K_{55}-K_{35}K_{54})+K_{45}(K_{54}K_1^{*\hat{i}}-K_{34}K_5^{*\hat{i}})+K_{44}(K_{35}K_5^{*\hat{i}}-K_{55}K_1^{*\hat{i}})]}{K_{35}(K_{43}K_{54}-K_{44}K_{53})+K_{34}(K_{45}K_{53}-K_{43}K_{55})+K_{33}(K_{44}K_{55}-K_{45}K_{54})},\quad G_2^{\sigma\hat{i}}=\frac{K_2^{2\hat{i}}}{K_{11}-K_{12}}$$

$$G_3^{\sigma\hat{i}}=\frac{K_3^{\hat{i}}(K_3^{\hat{i}}(K_{11}-K_{12})K_{16}+K_{26}((\dot{K}_{11}+2\dot{K}_{12})K_2^{\hat{i}}-(K_{11}-K_{12})K_4^{\hat{i}}))}{(K_{11}-K_{12})(K_{26}(\dot{K}_{11}+2\dot{K}_{12})-(K_{11}-K_{12})K_{66})}$$

$$G_4^{D\hat{i}}=\frac{K_4^{\hat{i}}(K_4^{*\hat{i}}(K_{35}K_{53}-K_{33}K_{55})+K_{45}(K_{33}K_5^{*\hat{i}}-K_{53}K_1^{*\hat{i}})+K_{43}(K_{55}K_1^{*\hat{i}}-K_{35}K_5^{*\hat{i}}))}{K_{35}(K_{43}K_{54}-K_{44}K_{53})+K_{34}(K_{45}K_{53}-K_{43}K_{55})+K_{33}(K_{44}K_{55}-K_{45}K_{54})}$$

$$G_5^{B\hat{i}}=\frac{K_5^{\hat{i}}[K_4^{*\hat{i}}(K_{33}K_{54}-K_{34}K_{53})+K_{44}(K_{53}K_1^{*\hat{i}}-K_{33}K_5^{*\hat{i}})+K_{43}(K_{34}K_5^{*\hat{i}}-K_{54}K_1^{*\hat{i}})]}{K_{35}(K_{43}K_{54}-K_{44}K_{53})+K_{34}(K_{45}K_{53}-K_{43}K_{55})+K_{33}(K_{44}K_{55}-K_{45}K_{54})}$$

$$G_6^{\vartheta\hat{i}}=\frac{K_6^{\hat{i}}[(\dot{K}_{11}+2\dot{K}_{12})(K_2^{\hat{i}}+K_3^{\hat{i}})-(K_{11}-K_{12})K_6^{\hat{i}}]}{(K_{11}-K_{12})K_{66}-K_{16}(\dot{K}_{11}+2\dot{K}_{12})}$$

上式中其它参数表达式见附录. 根据应变能强度理论 [21,22], 广义应变能强度因子可定义为

$$S_{\hat{i}}|_{\theta}=K_{\hat{i}n}^2\Big(\frac{a_{\hat{i}3n}^2}{E}+\frac{a_{\hat{i}1n}^2+a_{\hat{i}2n}^2}{\mu}+\frac{a_{\hat{i}4n}^2+a_{\hat{i}5n}^2+a_{\hat{i}6n}^2}{E'}\Big) \quad (17)$$

$a_{\hat{i}mn}$ 和材料有关参数表达式见附录. $E(E')$, μ 分别表示杨氏模量 (等效杨氏模量) 、剪切模量比.

3 数值方法

从超奇异积分方程组形式可看出, 最难处理的部分为超奇异积分部分, 核函数 $K_{ij}^{\hat{i}}$ 为 Gauss-Chebyshev 型积分, 可通过通常意义下的积分求解.

广义位移间断未知函数可表示为

$$\tilde{U}_1^{\hat{i}}(\xi_1^{\hat{i}},\xi_2^{\hat{i}})=F_J^{\hat{i}}(\xi_1^{\hat{i}},\xi_2^{\hat{i}})\xi_2^{\hat{i}\lambda_J}W^{\hat{i}}(\xi_1^{\hat{i}},\xi_2^{\hat{i}}) \quad (18)$$

其中 $W^{\hat{i}}(\xi_1^{\hat{i}},\xi_2^{\hat{i}})=\sqrt{(a^2-\xi_1^{\hat{i}2})(2b-\xi_2^{\hat{i}})}$, $F_J^{\hat{i}}(\xi_1^{\hat{i}},\xi_2^{\hat{i}})=\sum\limits_{s=0}^{S}\sum\limits_{h=0}^{H}a_{Jsh}^{\hat{i}}\xi_1^{s\hat{i}}\xi_2^{h\hat{i}}$. 把上式代入超奇异积分方程组, 可得到如下代数方程组

$$\sum_{\hat{i}}^{n}\sum_{s=0}^{S}\sum_{h=0}^{H}a_{Ish}^{\hat{i}}(I_{IJsh}^{\hat{i}1}+I_{IJsh}^{\hat{i}2}+I_{IJsh}^{\hat{i}3})=-p_J^{\hat{i}}(x_1^{\hat{i}},x_2^{\hat{i}}) \quad (19)$$

其中 $I_{IJsh}^{\hat{i}i}$ 具体表达式为

$$I_{11sh}^{\hat{i}1}=\int_0^{2\pi}[K_{11}(1-3\sin^2\theta^{\hat{i}})+K_{12}(1-3\cos^2\theta^{\hat{i}})]\Big(D_1^{\hat{i}1}\ln R-R^{-1}D_0^{\hat{i}1}+\int_0^R D_2^{\hat{i}1}\mathrm{d}r_1\Big)\mathrm{d}\theta^{\hat{i}} \quad (20)$$

$$I_{12sh}^{\hat{i}1}=\int_0^{2\pi}1.5(K_{11}+K_{12})\sin 2\theta^{\hat{i}}\Big(D_1^{\hat{i}2}\ln R-R^{-1}D_0^{\hat{i}2}+\int_0^R D_2^{\hat{i}2}\mathrm{d}r_1\Big)\mathrm{d}\theta^{\hat{i}} \quad (21)$$

$$I_{21sh}^{\hat{i}1}=\int_0^{2\pi}1.5(K_{11}+K_{12})\sin 2\theta^{\hat{i}}\Big(D_1^{\hat{i}1}\ln R-R^{-1}D_0^{\hat{i}1}+\int_0^R D_2^{\hat{i}1}\mathrm{d}r_1\Big)\mathrm{d}\theta^{\hat{i}} \quad (22)$$

$$I_{22sh}^{\hat{i}1}=\int_0^{2\pi}[K_{11}(1-3\cos^2\theta^{\hat{i}})+K_{12}(1-3\sin^2\theta^{\hat{i}})]\Big(D_1^{\hat{i}2}\ln R-R^{-1}D_0^{\hat{i}2}+\int_0^R D_2^{\hat{i}2}\mathrm{d}r_1\Big)\mathrm{d}\theta^{\hat{i}} \quad (23)$$

$$I_{61sh}^{\hat{i}1}=\int_0^{2\pi}[\dot{K}_{11}(1-3\cos^2\theta^{\hat{i}})-0.5\dot{K}_{12}\sin 2\theta^{\hat{i}}]\Big(D_1^{\hat{i}1}\ln R-R^{-1}D_0^{\hat{i}1}+\int_0^R D_2^{\hat{i}1}\mathrm{d}r_1\Big)\mathrm{d}\theta^{\hat{i}} \quad (24)$$

$$I_{\alpha 6sh}^{\hat{i}1}=\int_0^{2\pi}3(K_{16}\cos\theta^{\hat{i}}\delta_{1\alpha}+K_{26}\sin\theta^{\hat{i}}\delta_{2\alpha})\Big(D_1^{\hat{i}6}\ln R-R^{-1}D_0^{\hat{i}6}+\int_0^R D_2^{\hat{i}6}\mathrm{d}r_1\Big)\mathrm{d}\theta^{\hat{i}} \quad (25)$$

$$I_{62sh}^{\hat{i}1}=\int_0^{2\pi}[\dot{K}_{12}(1-3\sin^2\theta^{\hat{i}})-\dot{K}_{11}\cos\theta^{\hat{i}}\sin\theta^{\hat{i}}]\Big(D_1^{\hat{i}2}\ln R-R^{-1}D_0^{\hat{i}2}+\int_0^R D_2^{\hat{i}2}\mathrm{d}r_1\Big)\mathrm{d}\theta^{\hat{i}} \quad (26)$$

$$I_{66sh}^{\hat{i}1} = \int_0^{2\pi} 3K_{66}(\lambda_{31}\cos\theta^{\hat{i}} + \lambda_{32}\sin\theta^{\hat{i}})\left(D_1^{\hat{i}6}\ln R - R^{-1}D_0^{\hat{i}6} + \int_0^R D_2^{\hat{i}6}\mathrm{d}r_1\right)\mathrm{d}\theta^{\hat{i}} \tag{27}$$

$$I_{m6sh}^{\hat{i}1} = \int_0^{2\pi} 3\dot{K}_{m6}(\lambda_{31}\cos\theta^{\hat{i}} + \lambda_{32}\sin\theta^{\hat{i}})\left(D_1^{\hat{i}6}\ln R - R^{-1}D_0^{\hat{i}6} + \int_0^R D_2^{\hat{i}6}\mathrm{d}r_1\right)\mathrm{d}\theta^{\hat{i}} \tag{28}$$

$$I_{m\alpha sh}^{\hat{i}1} = \int_0^{2\pi} K_{m1}\cos\theta^{\hat{i}}\left(D_1^{\hat{i}\alpha}\ln R - R^{-1}D_0^{\hat{i}\alpha} + \int_0^R D_2^{\hat{i}\alpha}\mathrm{d}r_1\right)\mathrm{d}\theta^{\hat{i}} \tag{29}$$

$$I_{mnsh}^{\hat{i}1} = \int_0^{2\pi} K_{mn}\left(D_1^{\hat{i}n}\ln R - R^{-1}D_0^{\hat{i}n} + \int_0^R D_2^{\hat{i}n}\mathrm{d}r_1\right)\mathrm{d}\theta^{\hat{i}} \tag{30}$$

上式中其它参数具体表达式见附录.

4 数值结果及讨论

设磁电热弹耦合材料内含三维二裂纹在无限远处受到力载荷 $\sigma_{3i}^{1\infty}, \sigma_{3i}^{2\infty}$, 电载荷 $D_{33}^{1\infty}, D_{33}^{2\infty}$, 磁载荷 $B_{33}^{1\infty}, B_{33}^{2\infty}$, 热载荷 $\vartheta_{33}^{1\infty}, \vartheta_{33}^{2\infty}$ 共同作用. 无量纲化后的材料参数见文献 [41] 中表 2. 为计算和比较方便, 对广义 (应力、应变能) 强度因子进行无量纲化，无量化广义 (应力、应变能) 强度因子可表示为

$$F_{I,\lambda}^{\hat{i}} = K_{I,\lambda}^{\hat{i}}/\sigma_{3I}^{\hat{i}\infty}b^{1-\lambda}, \quad F_I^{\hat{i}} = K_I^{\hat{i}}/\sigma_{3I}^{\hat{i}\infty}\sqrt{b} \tag{31}$$

$$S_{\hat{i},\lambda}|_\theta = F_{n,\lambda}^2\left(\frac{a_{\hat{i}3n}^2}{E} + \frac{a_{\hat{i}1n}^2 + a_{\hat{i}2n}^2}{\mu} + \frac{a_{\hat{i}4n}^2 + a_{\hat{i}5n}^2 + a_{\hat{i}6n}^2}{E'}\right)S_{\hat{i}}|_\theta = F_n^2\left(\frac{a_{\hat{i}3n}^2}{E} + \frac{a_{\hat{i}1n}^2 + a_{\hat{i}2n}^2}{\mu} + \frac{a_{\hat{i}4n}^2 + a_{\hat{i}5n}^2 + a_{\hat{i}6n}^2}{E'}\right) \tag{32}$$

4.1 收敛性

在裂纹形状比 $a/b = 1$，裂纹间垂直距离比 $h/a = 1$，无限远处广义载荷为 $\sigma_{3i}^{1\infty}$ 时，配置点 $KK = LL = 20 \times 20$；多项式指数 $M = N = 13$, 无量纲广义应力强度因子随裂纹位置变化规律如表 1～ 表 3 所示.

表 1　在 $x_2 = 0$ 边，无量纲广义应力强度因子 $F_{1,\lambda}^1$, $F_{1,\lambda}^2$ 随多项式指数变化规律

Table 1　Convergence of dimensionless E-SIFs $F_{1,\lambda}^1$, $F_{1,\lambda}^2$ along $x_2 = 0$ with increasing the polynomial exponents

x_1/a	0/11	1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11
$F_{1,\lambda}^1$	0.6932	0.6915	0.6866	0.6783	0.6663	0.6501	0.6282	0.5990	0.5604	0.5062	0.4133
$F_{1,\lambda}^2$	0.02460	0.02454	0.02436	0.02407	0.02365	0.02307	0.02229	0.02126	0.01989	0.01796	0.01467

表 2　在 $x_1 = \pm a$ 边，无量纲广义应力强度因子 $F_{2,\lambda}^1$, $F_{2,\lambda}^2$ 随多项式指数变化规律

Table 2　Convergence of dimensionless E-SIFs, $F_{2,\lambda}^1$, $F_{2,\lambda}^2$ along $x_1 = \pm a$ with increasing the polynomial exponents

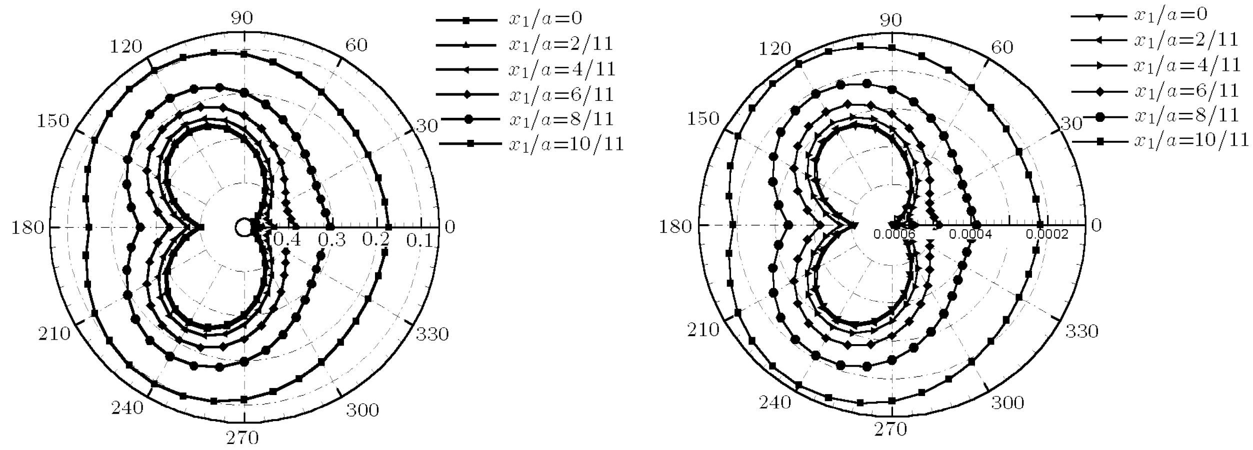
x_1/a	10/11	9/11	8/11	7/11	6/11	5/11	4/11	3/11	2/11	1/11	0/11
$F_{2,\lambda}^1$	0.5097	0.6049	0.6652	0.7080	0.7394	0.7625	0.7799	0.7926	0.8012	0.8060	0.8077
$F_{2,\lambda}^2$	0.01811	0.02149	0.02364	0.02516	0.02627	0.02710	0.02771	0.02816	0.02847	0.02864	0.02870

表 3　在 $x_2 = 0, 2b$ 边，无量纲广义应力强度因子 $F_{3,\lambda}^1$, $F_{3,\lambda}^2$ 随多项式指数变化规律

Table 3　Convergence of dimensionless E-SIFs $F_{3,\lambda}^1$, $F_{3,\lambda}^2$ along $x_2 = 0, 2b$ with increasing the polynomial exponents

x_1/a	10/11	9/11	8/11	7/11	6/11	5/11	4/11	3/11	2/11	1/11	0/11
$F_{3,\lambda}^1$	0.9109	0.9088	0.9023	0.8909	0.8744	0.8520	0.8227	0.7842	0.7338	0.6660	0.5537
$F_{3,\lambda}^2$	0.0324	0.03233	0.03210	0.03169	0.03110	0.03031	0.02926	0.02790	0.02610	0.02369	0.01970

应变能强度因子变化规律如图 2 所示，$S_{\alpha,\lambda}|_\theta$ 随 x_1/a 呈对称分布，在 $x_1/a = 0$ 处达到极值；在 $180° \sim 360°$ 间变化规律与 $0° \sim 180°$ 间变化规律呈对称分布，这与实际情况相吻合 [20~23].



(a) $S_{1,\lambda}|_\theta$ (b) $S_{2,\lambda}|_\theta$

图 2 裂纹 I 和 II 上广义无量纲应变能强度因子 $S_{\alpha,\lambda}|_\theta$ 随 θ 和 x_1/a 变化规律 ($KK=LL=20\times 20$, $M=N=13$)

Fig.2 Dimensional extended strain energy density factors $S_{\alpha,s}|_\theta$ varying with θ and x_1/a when $KK=LL=20\times 20$, $M=N=13$

4.2 一般情况

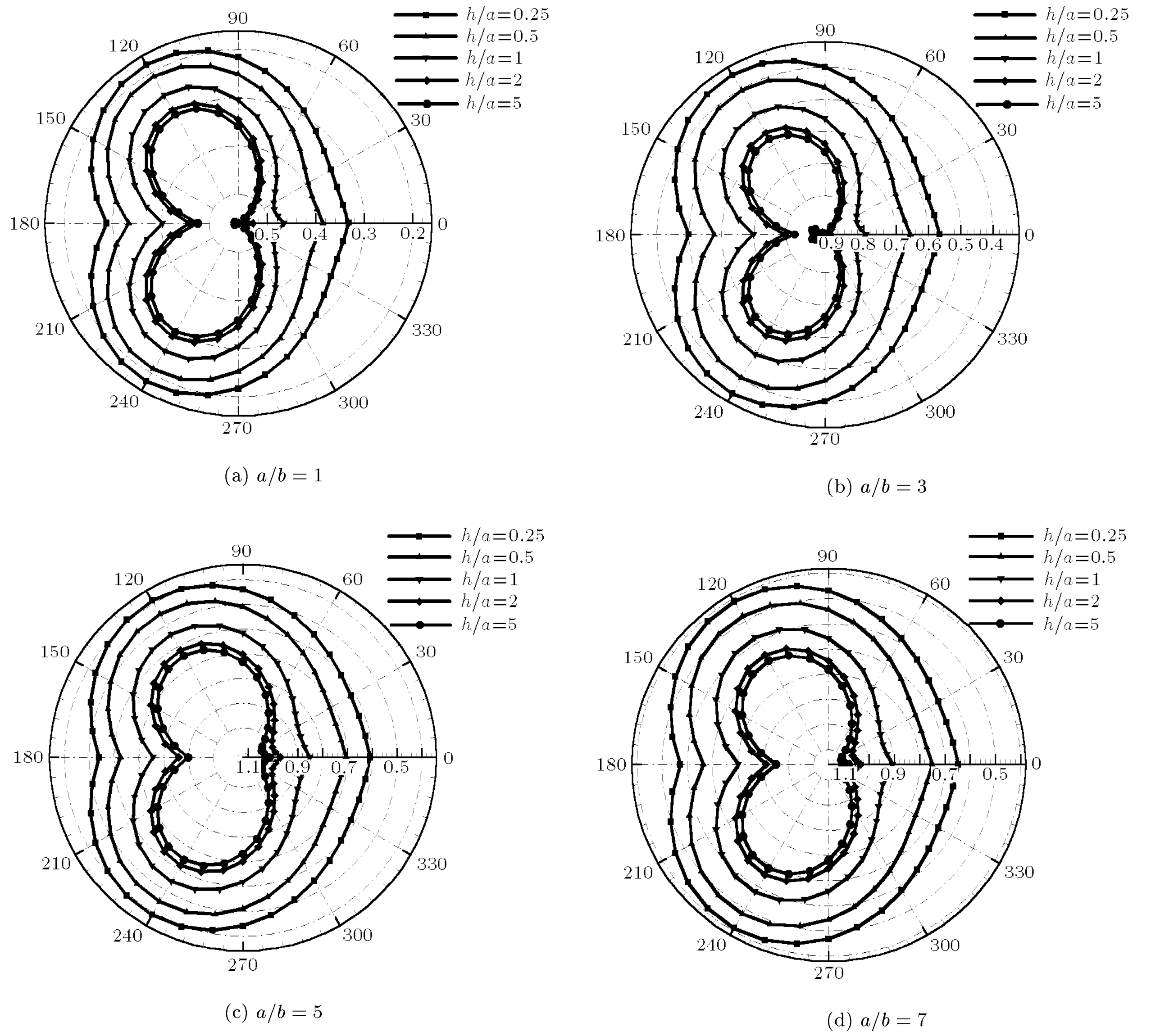
裂纹形状比 (b/a) 、裂纹面间距离比 (h/a) 对广义应力强度因子的影响如表 4 所示. 裂纹 I 上应力强度因子 F_I^1 随裂纹面间距离比增加而增大，裂纹 II 上应力强度因子 F_I^2 随裂纹面间距离比增加而减小；当 $h/a=5$ 时， F_I^1 收敛到某固定值 (单裂纹情况)， F_I^2 收敛到 0；结果显示裂纹间干扰和屏蔽作用随裂纹面间距离比增加而不断减弱，当 $h/a\geq 5$ 时，相互作用可忽略不计. 从数值结果还可看出，裂纹面上应力强度因子 F_I^α 均随裂纹形状比增加而增加，且增加梯度不断减少；当 $b/a=7$ 时， F_I^1, F_I^2 收敛到某固定值 (平面裂纹情况)，在 $b/a\geq 7$ 时，已经可以看作平面问题了.

表 4 无量纲最大广义应力强度因子 F_I^1, F_I^2 随裂纹形状比及裂纹面间距离比变化规律

Table 4 The maximum dimensionless F_I^1 and F_I^2 varying with the vertical spacing between two parallel flaws and the shape of flaw

b/a	h/a	F_1^1	F_2^1	F_3^1	F_1^2	F_2^2	F_3^2
1	0.5	0.630 4	0.734 5	0.828 4	0.071 7	0.083 5	0.094 2
	1.0	0.693 2	0.807 7	0.910 9	0.024 6	0.028 7	0.032 4
	2.0	0.739 4	0.861 5	0.971 6	0.004 9	0.005 7	0.006 4
	5.0	0.753 6	0.878 1	0.990 3	0.000 10	0.000 12	0.000 14
3	0.5	0.776 2	0.852 5	1.178 1	0.088 3	0.096 9	0.134 0
	1.0	0.853 5	0.937 4	1.295 5	0.030 3	0.033 3	0.046 1
	2.0	0.910 4	0.999 9	1.381 8	0.006 0	0.006 6	0.009 1
	5.0	0.927 9	1.019 1	1.408 4	0.000 14	0.000 16	0.000 22
5	0.5	0.774 7	0.863 4	1.253 3	0.088 1	0.098 2	0.142 5
	1.0	0.851 9	0.949 4	1.378 2	0.030 3	0.033 8	0.049 0
	2.0	0.908 6	1.012 7	1.470 0	0.006 0	0.006 7	0.009 7
	5.0	0.937 9	1.032 2	1.498 3	0.000 15	0.000 16	0.000 24

图 3, 图 4 表示最大无量纲应变能强度因子 $S_\alpha|_\theta$ 随裂纹形状比、裂纹面间距离比变化规律. $S_1|_\theta$ 随裂纹面间距离比增加而增大， $S_2|_\theta$ 随裂纹面间距离比增加而减小；当 $h/a=5$ 时， $S_1|_\theta$ 收敛到某固定值 (单裂纹情况)， $S_2|_\theta$ 则收敛到 0. 当 $h/a\geq 5$ 时，相互作用可忽略不计； $S_\alpha|_\theta$ 随裂纹形状比增加而增加，当 $b/a=7$ 时，收敛到一固定值 (平面裂纹情况)，变化规律与实际情况相吻合 [20~23].



(a) $a/b=1$　(b) $a/b=3$

(c) $a/b=5$　(d) $a/b=7$

图 3　裂纹 I 上广义无量纲应变能强度因子随 θ, a/b 和 h/a 变化规律 ($KK=LL=20\times 20$, $M=N=13$)

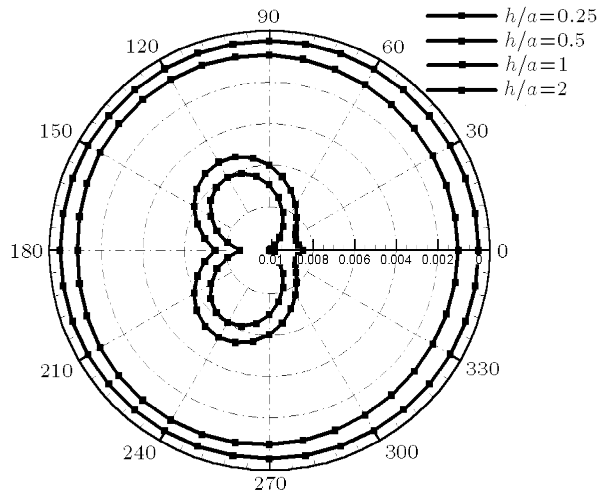
Fig.3　Dimensionless extended strain energy density factors on the 1st flaw varying with θ, a/b and h/a when $KK=LL=20\times 20$, $M=N=13$

5　讨　论

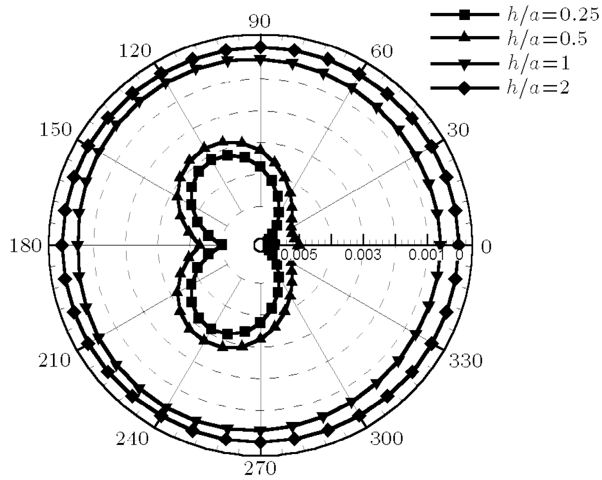
本文应用超奇异积分方程方法，分析了多场 (磁、电、热、弹) 载荷共同作用下的磁电热弹耦合材料内含三维多裂纹问题，主要结论如下：

(1) 多场 (磁、电、热、弹) 载荷共同作用下磁电热弹耦合材料内含三维多裂纹问题被转化为求解一以广义位移间断为未知函数的超奇异积分方程组问题. 在此基础上，通过庞杂的数学推导，得到了广义奇异应力场解析表达式，定义了广义 (应力和应变能) 强度因子和广义能量释放率.

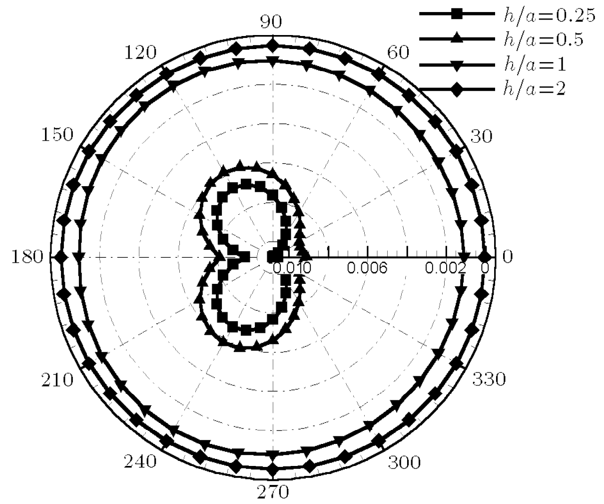
(2) 通过对超奇异积分方程组中各类奇异积分的专门数值处理，将该方程组转化成为一代数方程组，建立了超奇异积分方程组的数值方法，推导出广义 (应力和应变能) 强度因子数值计算公式. 以三维双裂纹为例，用 FORTRAN 语言编写了专门程序，计算了典型算例. 数值结果显示：本方法具有良好的收敛性和精度. 通过分析广义 (应力和应变能) 强度因子与裂纹形状比、裂纹面间距离比、广义载荷及材料参数之间的变化规律，得到裂纹间干扰和屏蔽作用不但与裂纹形状、位置及裂纹面距离有关，而且与载荷类型及材料参数有关.



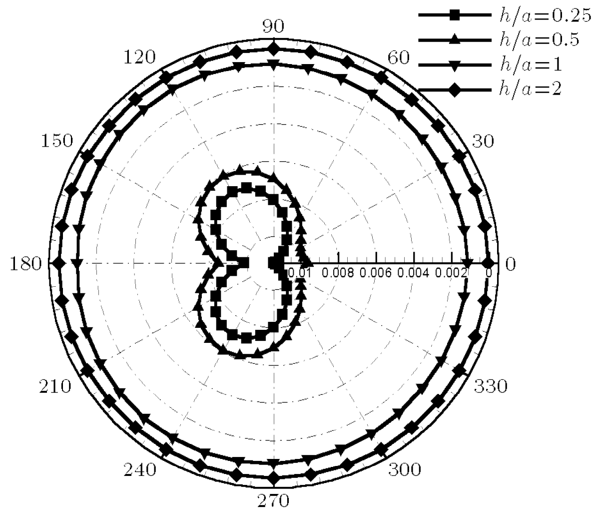
(a) $a/b=1$



(b) $a/b=3$



(c) $a/b=5$



(d) $a/b=7$

图 4 裂纹 II 上广义无量纲应变能强度因子随 θ, a/b 和 h/a 变化规律 ($KK=LL=20\times 20$, $M=N=13$)

Fig.4 Dimensionless extended strain energy density factors on the 2nd flaw varying with θ, a/b and h/a when $KK=LL=20\times 20$, $M=N=13$

(3) 对于工程中遇到的多场耦合作用下的复合材料内含多裂纹问题，可以应用本文提供的裂纹评定准则来分析和研究结构的可靠性. 在应力集中区域内，多裂纹间干扰和屏蔽作用对材料结构强度和可靠性的影响十分明显，在工程应用中应予以考虑.

参 考 文 献

1 Jordan NF, Eringen AC. On the static nonlinear theory of electromagnetic thermoelastic solids——I. *International Journal of Engineering Science*, 1964, 2(1): 59~95

2 Jordan NF, Eringen AC. On the static nonlinear theory of electromagnetic thermoelastic solids——II. *International Journal of Engineering Science*, 1964, 2(1): 97~114

3 Davis PM. Piezomagnetic computation of magnetic-anomalies due to ground loading by a man-made lake. *Pure and Applied Geophysics*, 1974, 112(5): 811~819

4 Tinkham M. Electromagnetic properties of superconductors. *Reviews of Modern Physics*, 1974, 46(4): 587~596

5 Vandenboomgaard J, Vanrun AMJG, Vansuchtelen J. Piezoelectric-piezomagnetic composites with magnetoelectric effect. *Ferroelectrics*, 1976, 14(1-2): 727~728

6 Vandenboomgaard J, Vanrun AMJG, Vansuchtelen J. Magnetoelectricity in piezoelectric-magnetostrictive composites. *Ferroelectrics*, 1976, 10(1-4): 295~298

7 Liu TJC, Chue CH. On the singularities in a bimaterial magneto-electro-elastic composite wedge under antiplane deformation. *Composite Structures*, 2006, 72(2): 254~265

8 Tan P, Tong LY. Modeling for the electro-magneto-thermo-

elastic properties of piezoelectric-magnetic fiber reinforced composites. *Composites Part a-Applied Science and Manufacturing*, 2002, 33(5): 631～645

9 Aboudi J. Micromechanical analysis of fully coupled electro-magneto-thermo-elastic multiphase composites. *Smart Materials & Structures*, 2001, 10(5): 867～877

10 Gao CF, Noda N. Thermal-induced interfacial cracking of magnetoelectroelastic material. *International Journal of Engineering Science*, 2004, 42(13-14): 1347～1360

11 Gao CF, Kessler H, Balke H. Fracture analysis of electromagnetic thermoelastic solids. *European Journal of Mechanics a-Solids*, 2003, 22(3): 433～442

12 Zhou YH, Zheng XJ. A general expression of magnetic force for soft ferromagnetic plates in complex magnetic fields. *International Journal of Engineering Science*, 1997, 35(15): 1405～1417

13 Zhou YH, Zheng XJ. A generalized variational principle and theoretical model for magnetoelastic interaction of ferromagnetic bodies. *Science in China* (Series A), 1999, 42(6): 618～626

14 Zheng XJ, Liu XE. A nonlinear constitutive model for Terfenol-D rods. *Journal of Applied Physics*, 2005, 97(053901)

15 Zheng XJ, Sun L. A nonlinear constitutive model of magneto-thermo- mechanical coupling for giant magnetostrictive materials. *Journal of Applied Physics*, 2006, 100(063906)

16 Zheng XJ, Wang XZ. A magnetoelastic theoretical model for soft ferromagnetic shell in magnetic field. *International Journal of Solids and Structures* 2003, 40: 6897～6912

17 Zhou YH, Gao YW, Zheng XJ. Buckling and post-buckling analysis for magneto-elastic-plastic ferromagnetic beam-plates with unmovable simple supports. *International Journal of Solids and Structures*, 2003, 40: 2875～2887

18 Zhou YH, Yang XB. Numerical simulations of thermomagnetic instability in high superconductors: Dependence on sweep rate and ambient temperature. *Physical Review B*, 2006, 74(054507)

19 Zhou YH, Zhao XF. Dynamical analysis of superconducting levitation with hysteresis. *Physical C*, 2006, 442(1): 55～62

20 Sih GC, Chen EP. Moving cracks in a finite strip under tearing action. *Journal of the Franklin Institute*, 1970, 290(1): 25～35

21 Sih GC. A three-dimensional strain energy density theory of crack propagation: three-dimensional of crack problems. Mechanics of Fracture. Sih GC, ed. Noordhoff International Publishing, Leyden 1975. II: xxv-xxvi

22 Sih GC. A Special Theory of Crack Propagation: Methods of Analysis and Solutions of Crack Problems. Mechanics of Fracture. Sih GC, ed. Noordhoff International Publishing, Leyden 1973, I: xxv-xxvii

23 Sih GC. Boundary problems for longitudinal shear cracks. Proceedings of Second Conference on Theoretical and Applied Mechanics, New York: Pergamon 1964. 117

24 Sih GC, Song ZF. Magnetic and electric poling effects associated with crack growth in $BaTiO_3$-$CoFe_2O_4$ composite. *Theoretical and Applied Fracture Mechanics*, 2003, 39(3): 209～227

25 杨卫. 宏微观断裂力学. 北京：国防工业出版社，1995(Yang Wei. Fracture Mechanisms of Macro-micro Mechanics. Beijing: National Deference Industry Press, 1995(in Chinse))

26 杨卫．力电失效学．北京：清华大学出版社，2001(Yang Wei. Mechatronic Reliability. Beijing: Tsinghua University Press, 2001 (in Chinese))

27 Li JY, Rogan RC, Üstündag E, Bhattacharya K. Domain switching in polycrystalline ferroelectric ceramics. Nature Materials, 2005, 4(10): 776～781

28 刘又文，方棋洪，蒋持平．压电螺位错与含界面裂纹圆形涂层夹杂的干涉．力学学报，2006, 38(2): 185～190 (Liu Youwen, Fang Qihong, Jiang Chiping. Electroelastic interaction between a piezoelectric screw dislocation and coated inclusion with interfacial cracks. *Chinese Journal of Theoretical and Applied Mechanics*, 2006, 38(2): 185～190(in Chinese))

29 Jones R, Peng D, Pitt S. Assessment of multiple flat elliptical cracks with interactions. *Theoretical and Applied Fracture Mechanics*, 2002, 38(3): 281～291

30 Martin-Meizoso A, Martinez-Esnaola JM, Fuentes-Perez M. Interactive effect of multiple crack growth on fatigue. *Theoretical and Applied Fracture Mechanics*, 1995, 23(3): 219～233

31 Kaw AK, Gadi KS. Mechanics of fiber-reinforced composites with doubly periodic matrix cracks. *Theoretical and Applied Fracture Mechanics*, 1993, 19(3): 173～182

32 Ma H, Zhao LG, Chen YH. Crack initiation angles for systems of two interacting cracks in media with rectilinear preferred directions. *Theoretical and Applied Fracture Mechanics*, 1996, 24(3): 253～261

33 Gdoutos EE. Stable growth of a crack interacting with a circular inclusion. *Theoretical and Applied Fracture Mechanics*, 1985, 3(2): 141～150

34 Gdoutos EE. Growth predictions of two interacting cracks. *Theoretical and Applied Fracture Mechanics*, 1984, 1(2): 133～137

35 Sih GC, Li CT. Initiation and growth characterization of corner cracks near circular hole. *Theoretical and Applied Fracture Mechanics*, 1990, 13(1): 69～80

36 Kassir MK, Sih GC. Three-dimensional stresses around elliptical cracks in transversely isotropic solids. *Engineering Fracture Mechanics*, 1968, 1(2): 327～345

37 Sih GC, Tang XS. Dual scaling damage model associated with weak singularity for macroscopic crack possessing a micro/mesoscopic notch tip. *Theoretical and Applied Fracture Mechanics*, 2004, 42(1): 1～24

38 Bojing Z, Taiyan Q. Application of hypersingular integral equation method to three-dimensional crack in electromagnetothermoelastic multiphase composites. *International Journal of Solids and Structures*, 2007, 44(18-19): 5994～6012

39 Qin TY, Tang RJ. Finite-part integral and boundary-element method to solve embedded planar crack problems. *International Journal of Fracture*, 1993, 60(4): 373~381

附录

$$K_{\alpha i}^{\hat{i}}=\boldsymbol{H}_{\alpha i}^{\hat{i}}\Big(R_{\hat{i}}^{-3}(c_{44}^2D_0s_0^2(\delta_{\bar{\alpha}\bar{\beta}}-3R_{\hat{i},\bar{\alpha}}R_{\hat{i},\bar{\beta}})+(\delta_{\alpha\beta}-3R_{\hat{i},\alpha}R_{\hat{i},\beta})\sum_{i=1}^{5}\rho_i^2t_i^2+3h^{2\hat{i}}R_{\hat{i}}^{-2}(s_0\delta_{\alpha\beta}-5R_{\hat{i},\alpha}R_{\hat{i},\beta}))\delta_{\beta i}+$$

$$\Big(3R_{\hat{i}}^{-4}R_{\hat{i},\alpha}\sum_{i=1}^{5}\lambda_{33}s_i^2t_i^1+3h^{2\hat{i}}R_{\hat{i},\alpha}^{-2}R_{\hat{i}}^{-2}(1-5s_0\delta_{\alpha\beta}-5h^{2\hat{i}}R_{\hat{i}}^{-2})\Big)\delta_{6i}\Big) \quad \text{(A1)}$$

$$K_{mi}^{\hat{i}}=\boldsymbol{H}_{mi}^{\hat{i}}\Big(\Big(R_{\hat{i}}^{-2}R_{\hat{i},\alpha}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1+3h^{2\hat{i}}R_{\hat{i},\alpha}^{-2}R_{\hat{i}}^{-2}(1-5s_0\delta_{\alpha\beta}-5h^{2\hat{i}}R_{\hat{i}}^{-2})\Big)\delta_{\alpha m}+$$

$$\Big(R_{\hat{i}}^{-3}\sum_{n=3}^{5}\sum_{i=1}^{5}\rho_i^m t_i^t+3h^{2\hat{i}}R_{\hat{i},\alpha}R_{\hat{i}}(1-5R_{\hat{i}}^2h_i^{2\hat{i}})\Big)\delta_{nm}+$$

$$\Big(3R_{\hat{i}}^{-4}\lambda_{3\alpha}R_{\hat{i},\alpha}\sum_{i=1}^{5}\lambda_i^{\nu}s_i^2\rho_i^m+3h^{2\hat{i}}R_{\hat{i},\alpha}^{-2}R_{\hat{i}}^{-2}(1-5s_0\delta_{\alpha\beta}-5h^{2\hat{i}}R_{\hat{i}}^{-2})\Big)\delta_{6m}\Big) \quad \text{(A2)}$$

$$K_{6i}^{\hat{i}}=\boldsymbol{H}_{6i}^{\hat{i}}\Big(R_{\hat{i}}^{-3}\Big(c_{44}^2D_0s_0^2(\delta_{\bar{\alpha}\bar{\beta}}-3R_{\hat{i},\bar{\alpha}}R_{\hat{i},\bar{\beta}})+(\delta_{\alpha\beta}-3R_{\hat{i},\alpha}R_{\hat{i},\beta})\sum_{i=1}^{5}\rho_i^2t_i^2+3h^{2\hat{i}}R_{\hat{i}}^{-2}(s_0\delta_{\alpha\beta}-5R_{\hat{i},\alpha}R_{\hat{i},\beta})\Big)\delta_{\beta i}+$$

$$\Big(3R_{\hat{i}}^{-4}R_{\hat{i},\alpha}\sum_{i=1}^{5}\lambda_{33}s_i^2t_i^1+3h^{2\hat{i}}R_{\hat{i},\alpha}^{-2}R_{\hat{i}}^{-2}(1-5s_0\delta_{\alpha\beta}-5h^{2\hat{i}}R_{\hat{i}}^{-2})\Big)\delta_{6i}\Big) \quad \text{(A3)}$$

$$K_{1i}^1=R^{-3}\Big(-3c_{44}^2D_0s_0^2R_{,2}R_{,1}+(1-3R_{,1}^2)\sum_{i=1}^{5}\rho_i^2t_i^2+3h^2R^{-2}(s_0-5R_{,1}^2)\Big)\delta_{1i}+R^{-3}\Big((1-3c_{44}^2D_0s_0^2R_{,2}^2)-$$

$$3R_{,1}R_{,2}\sum_{i=1}^{5}\rho_i^2t_i^2-15h^2R^{-2}R_{,1}R_{,2}\Big)\delta_{2i}+\Big(3R^{-4}R_{,1}\sum_{i=1}^{5}\lambda_{33}s_i^2t_i^1+3h^2R_{,1}^{-2}R^{-2}(1-5s_0\delta_{1\beta}-5h^2R^{-2})\delta_{6i}\Big) \quad \text{(A4)}$$

$$K_{2i}^1=R^{-3}\Big(c_{44}^2D_0s_0^2(1-3R_{,2}^2)-3R_{,2}R_{,1}\sum_{i=1}^{5}\rho_i^2t_i^2-15h^2R^{-2}5R_{,2}R_{,1}\Big)\delta_{1i}+R^{-3}\Big(-c_{44}^2D_0s_0^2 3R_{,2}R_{,1}+$$

$$(1-3R_{,2}^2)\sum_{i=1}^{5}\rho_i^2t_i^2+3h^2R^{-2}(s_0-5R_{,2}^2)\Big)\delta_{2i}+\Big(3R^{-4}R_{,2}\sum_{i=1}^{5}\lambda_{33}s_i^2t_i^1+3h^2R_{,2}^{-2}R^{-2}(1-5s_0\delta_{22}-5h^2R^{-2})\delta_{6i}\Big) \quad \text{(A5)}$$

$$K_{3i}^1=\Big(R^{-2}R_{,1}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1+3h^2R_{,1}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\Big)\delta_{1i}+$$

$$\Big(R^{-2}R_{,2}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1+3h^2R_{,2}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\Big)\delta_{2i}+$$

$$\Big(2R^{-3}\sum_{n=3}^{5}\sum_{i=1}^{5}\rho_i^3t_i^t+3h^2R(R_{,1}+R_{,2})(1-5R^2h^2)\Big)\delta_{3i}+\Big(3R^{-4}\lambda_{31}R_{,1}\sum_{i=1}^{5}\lambda_i^{\nu}s_i^2\rho_i^3+3h^2R_{,1}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})+$$

$$3R^{-4}\lambda_{32}R_{,2}\sum_{i=1}^{5}\lambda_i^{\nu}s_i^2\rho_i^3+3h^2R_{,2}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\delta_{6i}\Big) \quad \text{(A6)}$$

$$K_{4i}^1=\Big(R^{-2}R_{,1}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1+3h^2R_{,1}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\Big)\delta_{1i}+$$

$$\left(R^{-2}R_{,2}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1+3h^2R_{,2}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\right)\delta_{2i}+$$
$$\left(2R^{-3}\sum_{n=3}^{5}\sum_{i=1}^{5}\rho_i^4t_i^t+3h^2R(R_{,1}+R_{,2})(1-5R^2h^2)\right)\delta_{4i}+\left(3R^{-4}\lambda_{31}R_{,1}\sum_{i=1}^{5}\lambda_i^{\nu}s_i^2\rho_i^4+3h^2R_{,1}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})+\right.$$
$$\left.3R^{-4}\lambda_{32}R_{,2}\sum_{i=1}^{5}\lambda_i^{\nu}s_i^2\rho_i^4+3h^2R_{,2}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\delta_{6i}\right) \tag{A7}$$

$$K_{5i}^1=\left(R^{-2}R_{,1}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1+3h^2R_{,1}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\right)\delta_{1i}+$$
$$\left(R^{-2}R_{,2}\sum_{i=1}^{5}A_i^{\Upsilon}t_i^2\rho_i^1+3h^2R_{,2}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\right)\delta_{2i}+$$
$$\left(2R^{-3}\sum_{n=3}^{5}\sum_{i=1}^{5}\rho_i^5t_i^t+3h^2R(R_{,1}+R_{,2})(1-5R^2h^2)\right)\delta_{5i}+\left(3R^{-4}\lambda_{31}R_{,1}\sum_{i=1}^{5}\lambda_i^{\nu}s_i^2\rho_i^5+3h^2R_{,1}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})+\right.$$
$$\left.3R^{-4}\lambda_{32}R_{,2}\sum_{i=1}^{5}\lambda_i^{\nu}s_i^2\rho_i^5+3h^2R_{,2}^{-2}R^{-2}(1-5s_0-5h^2R^{-2})\delta_{6i}\right) \tag{A8}$$

$$K_{6i}^1=R^{-3}\left(c_{44}^2D_0s_0^2(1-3R_{,2}^2)+(1-3R_{,1}^2)\sum_{i=1}^{5}\rho_i^2t_i^2+3h^2R^{-2}(s_0-5R_{,\beta}^2)\right)\delta_{1i}+R^{-3}\left(-3c_{44}^2D_0s_0^23R_{,1}R_{,2}-3R_{,2}R_{,1}\sum_{i=1}^{5}\rho_i^2t_i^2-\right.$$
$$\left.15h^2R^{-2}R_{,2}R_{,1}\right)+R^{-3}\left(c_{44}^2D_0s_0^2(1-3R_{,1}^2)+(1-3R_{,2}R_{,2}^2)\sum_{i=1}^{5}\rho_i^2t_i^2+3h^2R^{-2}(s_0-5R_{,2}^2)\right)\delta_{2i}+\left(3R^{-4}(R_{,1}+R_{,2})\times\right.$$
$$\left.\sum_{i=1}^{5}\lambda_{33}s_i^2t_i^1+(R_{,1}^{-2}+R_{,2}^{-2})3h^2R^{-2}(1-5s_0-5h^2R^{-2})\right)\delta_{6i} \tag{A9}$$

$$r=((\xi_1-x_1)^2+(\xi_2-x_2)^2)^{1/2},\quad r_{,\alpha}=(\xi_\alpha-x_\alpha)/r,\quad R_{i,\alpha}=(\xi_\alpha-x_\alpha)/R \tag{A10}$$

$$R_i=((\xi_1-x_1)^2+(\xi_2-x_2)^2+h^{2\hat{i}})^{1/2}K_{\alpha\beta}^1=K_{\alpha\beta}^2,\quad K_{\alpha m}^1=-K_{\alpha m}^2,\quad K_{\alpha6}^1=K_{\alpha6}^2,\quad K_{6i}^1=K_{6i}^2 \tag{A11}$$

$$R_i^{\hat{i}}=((x_1-\xi_1)^2+(x_2-\xi_2)^2+(z_i+\hat{i}h)^2)^{0.5},\quad \dot{R}_i=R_i+z_i(1+\hat{i}h) \tag{A12}$$

$$\hat{K}_2^{\hat{i}}=(K_3^{\hat{i}}(K_{11}-K_{12})K_{16}+K_{26}((\hat{K}_{11}+2\hat{K}_{12})K_2^{\hat{i}}-(K_{11}-K_{12})K_6^{\hat{i}}))/(K_{11}-K_{12})(K_{26}(\hat{K}_{11}+2\hat{K}_{12})-$$
$$(K_{11}-K_{12})K_{66}) \tag{A13}$$

$$K_1^{*\hat{i}}=K_1^{\hat{i}}+K_{31}\hat{K}_2^{\hat{i}}+K_{36}\hat{K}_6^{\hat{i}}K_5^{*\hat{i}}=K_5^{\hat{i}}+K_{51}\hat{K}_2^{\hat{i}}+K_{56}\hat{K}_6^{\hat{i}},\quad K_4^{*\hat{i}}=K_4^{\hat{i}}+K_{41}\hat{K}_2^{\hat{i}}+K_{46}\hat{K}_6^{\hat{i}} \tag{A14}$$

$$\hat{K}_6^{\hat{i}}=((\dot{K}_{11}+2\dot{K}_{12})(K_2^{\hat{i}}+K_3^{\hat{i}})-(K_{11}-K_{12})K_6^{\hat{i}})/((K_{11}-K_{12})K_{66}-K_{16}(\dot{K}_{11}+2\dot{K}_{12})) \tag{A15}$$

$$a_{\hat{i}1n}=\delta_{3n}r_{\hat{i}0}^{0.5}\cos\theta_{\hat{i}0}+\delta_{1n}\left(\sum_{i=1}^{5}A_i^{\Upsilon}r_i^{0.5}\cos\theta_{\hat{i}i}\cot2\theta_{\hat{i}i}(\iota_{33}\delta_{3n}+\varsigma_{33}\delta_{4n}+\eta_{33}\delta_{5n})+\rho_i^3s_{\hat{i}i}r_{\hat{i}i}^{-0.5}t_2\cot\theta_{\hat{i}i}\cos^{-1}\theta_{\hat{i}i}\right) \tag{A16}$$

$$a_{\hat{i}2n}=\sum_{i=1}^{5}r_{\hat{i}i}^{-0.5}\left[\rho_i^2\cos\theta_{\hat{i}i}(t_i^2\delta_{1n}+4t_i^3\delta_{2n}+4t_i^4\delta_{4n}+4t_i^5\delta_{5n})+\delta_{6n}\left(\lambda_{33}\left(15\lambda_{33}s_{\hat{i}i}^2\cot2\theta_{\hat{i}i}\left(3\cos\theta_{\hat{i}i}-3\sin^22\theta_{\hat{i}i}\cos\theta_{\hat{i}i}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{1}{4}\sin^22\theta_{\hat{i}i}\cos6\theta_{\hat{i}i}\right)-2\lambda_{33}r_{\hat{i}i}^{-2}\sin^{-1}2\theta_{\hat{i}i}^{i}+0.5\lambda_{32}r_{\hat{i}i}^{-1}\sin^{-1}\theta(2\cos^{-1}2\theta_{\hat{i}i}-3\cos2\theta_{\hat{i}i}-\cos10\theta_{\hat{i}i})\right)\right)\right] \tag{A17}$$

$$a_{\hat{i}3n}=\sum_{i=1}^{5}\left[\rho_i^3s_{\hat{i}i}\iota_{33}(A_i^{\Upsilon}r_{\hat{i}i}^{0.5}\cos\theta_{\hat{i}i}+6\cot\theta_{\hat{i}i})(t_i^3\delta_{2n}+t_i^4\delta_{4n}+t_i^5\delta_{5n})+\delta_{n1}(A_i^{\Upsilon}r_{\hat{i}i}^{0.5}\cos\theta_{\hat{i}i}\cot2\theta_{\hat{i}i}(\iota_{33}\delta_{3n}+\varsigma_{33}\delta_{4n}+\eta_{33}\delta_{5n})+\right.$$
$$\left.\rho_i^3s_{\hat{i}i}r_{\hat{i}i}^{-0.5}t_2\cot2\theta_{\hat{i}i}\cos^{-1}\theta_{\hat{i}i}+\delta_{n6}(\rho_i^3s_{\hat{i}i}^2\lambda_i^{\vartheta}r_{\hat{i}i}^{0.5}\cot2\theta_{\hat{i}i}\cos\theta_{\hat{i}i}+A_i^{\Upsilon}s_{\hat{i}i}^2\lambda_{33}\lambda_i^{\vartheta}r_{\hat{i}i}^{-0.5}\cos\theta_{\hat{i}i}(\iota_{33}\delta_{3n}+\varsigma_{33}\delta_{4n}+\eta_{33}\delta_{5n})))\right] \tag{A18}$$

$$a_{\hat{i}4n} = \sum_{i=1}^{5} \Big[\rho_i^4 s_{\hat{i}i}\zeta_{33}(A_i^{\Upsilon} r_{\hat{i}i}^{0.5}\cos\theta_{\hat{i}i} + 6\cot\theta_{\hat{i}i})(t_i^3\delta_{2n} + t_i^4\delta_{4n} + t_i^5\delta_{5n}) + \delta_{n1}(A_i^{\Upsilon} r_{\hat{i}i}^{0.5}\cos\theta_{\hat{i}i}\cot 2\theta_{\hat{i}i}(\iota_{33}\delta_{3n} + \varsigma_{33}\delta_{4n} + \eta_{33}\delta_{5n}) + \rho_i^4 s_{\hat{i}i} r_{\hat{i}i}^{-0.5} t_2 \cot 2\theta_{\hat{i}i}\cos^{-1}\theta_{\hat{i}i} + \delta_{n6}(\rho_i^4 s_{\hat{i}i}^2 \lambda_i^{\vartheta} r_{\hat{i}i}^{0.5}\cot 2\theta_{\hat{i}i}\cos\theta_{\hat{i}i} + A_i^{\Upsilon} s_{\hat{i}i}^2 \lambda_{33}\lambda_i^{\vartheta} r_{\hat{i}i}^{-0.5}\cos\theta_{\hat{i}i}(\iota_{33}\delta_{3n} + \varsigma_{33}\delta_{4n} + \eta_{33}\delta_{5n})))\Big] \tag{A19}$$

$$a_{\hat{i}5n} = \sum_{i=1}^{5} \Big[\rho_i^5 \eta_{33} s_{\hat{i}i}(A_i^{\Upsilon} r_{\hat{i}i}^{0.5}\cos\theta_{\hat{i}i} + 6\cot\theta_{\hat{i}i})(t_i^3\delta_{2n} + t_i^4\delta_{4n} + t_i^5\delta_{5n}) + \delta_{n1}(A_i^{\Upsilon} r_{\hat{i}i}^{0.5}\cos\theta_{\hat{i}i}\cot 2\theta_{\hat{i}i}(\iota_{33}\delta_{3n} + \varsigma_{33}\delta_{4n} + \eta_{33}\delta_{5n}) + {} + \rho_i^5 s_{\hat{i}i} r_{\hat{i}i}^{-0.5} t_2 \cot 2\theta_{\hat{i}i}\cos^{-1}\theta_{\hat{i}i} + \delta_{n6}(\rho_i^5 s_{\hat{i}i}^2 \lambda_i^{\vartheta} r_{\hat{i}i}^{0.5}\cot 2\theta_{\hat{i}i}\cos\theta_{\hat{i}i} + A_i^{\Upsilon} s_{\hat{i}i}^2 \lambda_{33}\lambda_i^{\vartheta} r_{\hat{i}i}^{-0.5}\cos\theta_{\hat{i}i}(\iota_{33}\delta_{3n} + \varsigma_{33}\delta_{4n} + \eta_{33}\delta_{5n})))\Big] \tag{A20}$$

$$a_{\hat{i}6n} = \sum_{i=1}^{5} A_i^{\Upsilon}\Big[r_{\hat{i}i}^{-0.5}\cos\theta_{\hat{i}i}(\lambda_{32} + 4\lambda_{33}s_{\hat{i}i}^2)(t_i^3\delta_{3n} + t_i^4\delta_{4n} + t_i^5\delta_{5n}) + \delta_{1n}(r_{\hat{i}i}^{-0.5}(\cos\theta_{\hat{i}i}(\lambda_{32} + 4\lambda_{33}s_{\hat{i}i}^2)t_2 + 4\lambda_{32}t_2 s_{\hat{i}i}\lambda_i^{\vartheta}(2\lambda_{32}\cos\theta_{\hat{i}i} + 3\lambda_{33} r_{\hat{i}i}^{-2}\sin^{-1}\theta_{\hat{i}i})/3) + \delta_{6n}45\lambda_{32}s_i^2\lambda_i^{\vartheta}\cot 2\theta_{\hat{i}i}\left(\cos\frac{\theta_{\hat{i}i}}{2} - \cos\theta_{\hat{i}i}\sin 2\theta_{\hat{i}i} - \frac{\sin^2\theta_{\hat{i}i}\cos 3\theta_{\hat{i}i}}{6}\right)(2s_{\hat{i}i}^2\lambda_{33} + \lambda_{32} r_{\hat{i}i}^{-1}\sin 2\theta_{\hat{i}i}))\Big] \tag{A21}$$

MULTIPLE THREE-DIMENSIONAL CRACKS IN FULLY COUPLED ELECTROMAGNETOTHERMOELASTIC MULTIPHASE COMPOSITES

Zhu Bojing[*,†] Qin Taiyan[*,1)]

*(*College of Science, China Agricultural University, Beijing* 100083, *China*)

†(*College of Earth Science, Graduate University of Chinese Academy of Sciences, Beijing* 100049, *China*)

Abstract This work presents the hypersingular integral equation method to analyze the multiple three-dimensional cracks problem in fully coupled electromagnetothermoelastic multiphase composites under extended electro-magneto-thermo-elastic coupled loading through intricate theoretical analysis and numerical simulations. First, the problem is reduced to solving a set of hypersingular integral equations. Analytical solutions for the extended singular stresses, the extended stress intensity factors, the extended strain energy factors and the extended energy release rate near the crack front are obtained, respectively. The numerical method for the hypersingular integral equations subjected to extended coupled loads is proposed. Finally, numerical solutions of the extended stress intensity factors and the extended strain energy factors for two interacting three-dimensional cracks are given, and the effect of cracks orientation, interaction and shielding is discussed

Key words multiple three-dimensional crack, electromagnetothermoelastic coupled multiphase composite, hypersingular integral equation, extended stress intensity factor

---

Received 13 June 2007, revised 24 September 2007.

1) E-mail: tyqin@cau.edu.cn