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RESEARCH PAPER

Improved suboptimal Bang–Bang control of aseismic buildings with variable friction dampers

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Abstract One of the challenges in civil engineering is to find an innovative means of suppressing the structural vibration due to earthquake and wind loadings. This paper presents an approach for effectively suppressing vibrations of a structure with variable friction damper using a new Bang–Bang control input. A continuous function of story velocities is used to represent the improved control to reduce chatter, high frequency switching and avoid instability. With a genetic algorithm, the amplitudes of control and preloading friction forces individually prescribed in the controller and damper are optimized for enhancing the seismic performance of buildings. The control strategy for the friction damper is proposed for a three story building with one variable friction damper installed at the first story for seismic reduction. The numerical results indicate that a better reduction of peak response accelerations of floors can be achieved than those of the unmodified controller, and the adaptability of the control system is also improved greatly by comparison with the reduction ratios of the structural response energy excited by different earthquake intensities.

Keywords Suboptimal Bang–Bang control · Aseismic building · Variable friction damper · Vibration reduction

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1 Introduction

As an effective approach to protect engineering structures from earthquake damage, the structural control is bringing about a revolution for the aseismic design of structures in civil engineering. Since the concept of the structural control in civil engineering was first proposed by Yao [1] in 1972, this field has made a great progress from theory to practice. To date, many newly developed control methods and energy dissipating devices have been applied to enhance the performance of structures under ground acceleration excitations.

The normal friction damper is a typical passive device used in an energy dissipation system and is widely used in civil engineering for vibration suppression for seismic structures [2,3]. It uses the solid friction that develops between two solid bodies sliding relative to one another to provide the desired energy dissipation. Like other passive energy dissipation devices, the passive friction damper cannot alter its mechanical parameters in real time according to structural responses or excitation information. Thus, it is impossible to achieve an optimal structural vibration control using the device. For example, when the contacting force is very large, the damper may not slide under a small or moderate earthquake and therefore will not dissipate any energy. On the other hand, when the contacting force is not large enough, the damper only dissipates a little amount of energy under a large earthquake due to its small sliding frictional force.

Thereby, it is desirable for friction dampers to have an adaptive ability to adjust the slip force in response to various levels of earthquake excitations. Semi-active control systems offer the ability to change the mechanical properties of dampers without requiring the associated large power sources, and combine the advantages of both passive and active control strategies, as an innovative, exciting and evolving technology. A variable friction damper is one type of semi-active energy dissipation devices. It is generally made by modifying a passive friction damper to allow the adjustment of the normal force and in its turn the sliding frictional force by an actuator. Kannan et al. [4] designed a semi-active friction damper with a hydraulic actuator. Its main disadvantage is that it can not rapidly modulate the actuation force for the contacting force to reach the required pressure and it may introduce the backlash effect when used. Agrawal et al. [5] proposed a novel semi-active electromagnetic friction damper, which regulates the friction force by the current in solenoids across the damper without any time delay. But it is limited to electromagnetic actuators as linear drivers. Although they can generate large force and displacement, the large size, weight, electrical demands and cost of these actuators make the device impractical [6]. Another approach to make a semi-active friction damper is the utilization of piezoelectric materials that can generate a significant amount of force in milliseconds under a constrained condition when exposed to an electric field. The most inconvenient factor for a piezoelectric actuator used to actively suppress vibrations of a seismic structures is its small endurable displacement. However, it is still a good natural candidate as an actuator to adjust the contacting force of semi-active friction dampers. Such systems were investigated for the seismic protection of civil engineering structures by Chen and Garrett et al. [7], Chen et al. [8], Ou et al. [9] and Li et al. [10]. Chen et al. [8] described preliminary design steps for a piezoelectric variable friction damper and designed a prototype to control a 1/4 scaled framed structural model. Ou et al. presented a T-shaped piezoelectric variable friction damper which consists of a piezoelectric stack actuator and a Pall friction damper. Li et al. [10] developed a new type of piezoelectric friction damper by combining the piezoelectric tube stack actuators with a slotted bolted connection. In terms of control methods for semi-active friction dampers, Inaudi [11] presented an algorithm which makes the contacting force between the sliding surfaces of the damper proportional to the absolute value of the prior local peak deformation of the damper and leads to a non-linear force-deformation relation that satisfies homogeneity of degree one. Lu [12] proposed a control method that may predict the minimal friction force required to keep the damper in its stick state and adjust its slip force to be slightly less than this value to maintain the damper in its sliding state to continuously dissipate energy within the duration of an earthquake of any intensity and waveform by assuming a friction damper in its sticking state at the next time step. Lu [13] also presented a semi-active modal control scheme for the online determination of the controllable clamping forces of a variable friction damper. The feedback gain of the above method was modified from that of an active modal control to accommodate the control force constraint of the variable friction damper by using a Heaviside function. Yang, He and Agrawal [14,15] improved the previous local peak deformation of friction dampers to increase the possibility of slippage in the damper at all levels of excitations, and to alleviate the possible chattering effect of the original controller by using a boundary layer around the switching of the controller for a hybrid controlled structure against near-fault earthquakes. Nashitani et al. [16] presented a semi-active scheme involving no structural modeling for control of a variable slip-force damper. The damper exhibits bilinear hysteresis with a ductile factor equal to two regardless of the level of the seismic excitation. Chen et al. [17] suggested a velocity and displacement feedback control method which combines the viscous and non-linear Reid damping mechanisms to effectively suppress the vibration of structures in a velocity-sensitive environment such as high-technology facilities against micro-vibration and civil infrastructure systems subjected to a pulse like a ground motion.

This paper presents a new control strategy based on the improved suboptimal Bang–Bang control algorithm that takes into account the features of friction dampers. Four different continuous functions [18] are introduced as velocities of stories where the variable friction dampers are installed, and only one is used in this paper to slow up the speedy switching and to reduce the chattering effect when the responses of the structure cross the zero point in the state space. In what follows, the basic principles of the variable friction damper and structural control are described in details. Then the derivation of the suboptimal Bang–Bang control method proposed by Wu et al. [19] and other modified methods are presented for a clear illustration of the technique embodied here. By the genetic algorithm (GA), the limit sliding force and preload applied on the sliding surface are optimized for the improvement of control performance. Finally, the influences of the parameters in the controller on the control effect are analyzed and the adaptive capability of this algorithm is tested by keeping the parameters fixed and adjusting the ground motion intensities in numerical analyses of a three-story structure with a variable friction damper installed at the first story.

2 Basic principles

Normally, it is difficult to analyze a structural system with a non-linear frictional damping characterized by discontinuous mathematical expressions. The simplest and most common friction model is known as the Coulomb friction, which is independent of loading frequency and is accurate enough for representing the mechanical properties of friction dampers used in civil engineering structures.

For a pure Coulomb friction damper, the controllable frictional-sliding force, $f(t)$, can be expressed as:

$$f(t) = -\mu N(t)\mathrm{sgn}(v) \quad \text{if } v \neq 0, \tag{1}$$

and

$$-\mu N(t) \leq f(t) \leq \mu N(t) \quad \text{if } v = 0, \tag{2}$$

where μ is the frictional coefficient; $N(t)$ is the normal force across the contacting surface; v is the sliding velocity of the damper; sgn is the signum function. These two equations depict the complete kinestates of the friction damper excited by dynamic loads. Equation (1) expresses the slip phase when the sliding velocity is nonzero, and Eq. (2) represents the sticking phase when the sliding velocity is zero. Assuming that the force produced by a friction damper is in the horizontal direction and the damper conforms to a rigid plastic model, while the sliding surfaces are stuck together, the absolute value of the control force, f_s, can be approximately written as follows:

$$f_s = |i + f_r| \quad \text{when } |f| \geq f_s \text{ and } v = 0, \tag{3}$$

where i is the inertial force applied on the mass of each story due to dynamic actions; f_r is the restoring force provided by the structural stiffness. Utilizing Eq. (3), the sticking-slip phase can be captured and the control force in the sticking phase is obtained by numerical analyses.

Considering a linear structural model with n degrees of freedom and m control forces, its equation of motion is expressed as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = -\boldsymbol{M}\boldsymbol{I}\ddot{x}_g(t) + \boldsymbol{H}\boldsymbol{u}(t), \tag{4}$$

where $\ddot{\boldsymbol{x}}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$ are n dimensional vectors representing the structural acceleration, velocity and displacement at time t, relative to the structural basement, respectively; $\ddot{x}_g(t)$ is the ground acceleration at the structural basement; $\boldsymbol{u}(t)$ is an m dimensional control force vector, whose components are control forces produced by control devices; $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$ represent $n \times n$ dimensional mass, stiffness, and damping matrices of the model, respectively; $\boldsymbol{I}$ denotes an n dimensional identity vector that indicates the DOF where the seismic excitation acts; $\boldsymbol{H}$ is an $n \times m$ dimensional matrix that represents the DOFs where the control forces act.

In the state space, Eq. (4) becomes

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}\ddot{x}_g(t), \tag{5}$$

where $\boldsymbol{z}(t) = \begin{Bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{Bmatrix}$, is a state space vector; $\boldsymbol{A} = \begin{bmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}$,

is an open loop system matrix;

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ \boldsymbol{M}^{-1}\boldsymbol{H} \end{bmatrix}; \quad \boldsymbol{E} = \begin{Bmatrix} 0 \\ -1 \end{Bmatrix}.$$

3 Modified Bang–Bang control law

The Bang–Bang control is a classic nonlinear control algorithm that can put the structure in a stable condition and avoid the saturation problem. This method can considerably reduce the peak response as compared to linear control methods. The optimal control force of the Bang–Bang control algorithm can be obtained by minimizing the objective index expressed as:

$$\boldsymbol{J} = \frac{1}{2}\int_0^{t_f} [\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{z}(t)]\mathrm{d}t \quad \text{s.t. } |\boldsymbol{u}(t)| \leq \boldsymbol{u}_{\max}, \tag{6}$$

where $\boldsymbol{Q}$ is a $2n \times 2n$ dimensional positive semi-definite weighting matrix. One of the problems in the real application of the Bang–Bang control law is that the resulting control force is governed by co-states rather than states. Another problem is that the co-states are solutions of a differential equation. The online calculation of the control force takes much time, and may introduce a time delay and instability to the control system. Furthermore, this algorithm requires the actuator to have the ability to response very fast, which produces undesirable control chattering near the origin of the state-space due to the high-speed switching of control forces between the two extreme values.

To solve these problems, the Bang–Bang control method was modified and the controller was improved by Wu et al. [19]. The whole derivation of the modified controller is repeated here as follows.

The control force is first expressed as a function of states instead of co-states to avoid the online calculation of the differential equation. Minimizing the performance index as a quadratic Lyapunov function expressed by Eq. (7), a suboptimal Bang–Bang control law can be derived as

$$\boldsymbol{V}(\boldsymbol{z}) = \boldsymbol{z}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}, \tag{7}$$

where $\boldsymbol{S}$ is a solution of the following Lyapunov matrix equation (8), where $\boldsymbol{Q}$ is selected to be a symmetric and positive semi-definite matrix:

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{S} + \boldsymbol{S}\boldsymbol{A} = -\boldsymbol{Q}. \tag{8}$$

The time derivative of the Lyapunov function is

$$\dot{\boldsymbol{V}}(\boldsymbol{z}) = \dot{\boldsymbol{z}}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z} + \boldsymbol{z}^{\mathrm{T}}\boldsymbol{S}\dot{\boldsymbol{z}}. \tag{9}$$

Substituting Eq. (5) into Eq. (9) and neglecting the term of earthquake excitation, we obtain

$$\dot{\boldsymbol{V}}(\boldsymbol{z}) = -\boldsymbol{z}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{z} + 2\boldsymbol{u}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}. \tag{10}$$

Obviously, if the control force takes the form:

$$u(t) = -u_{\max}\mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}(t)), \tag{11}$$

the time derivative of the Lyapunov function, i.e. Eq. (10), will take the minimum. The control law given by Eq. (11) is called a suboptimal Bang–Bang control method, by which the previous two problems have been solved successfully, but the high speedy switching and chattering problems still remain in the controller. Hence, further measures of a smooth switch from zero to other points in the state space need to be taken to improve the performance of the suboptimal controller for applications.

Define

$$r(t) = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}(t), \tag{12}$$

Eq. (11) can be rewritten as:

$$u(t) = -u_{\max}\frac{r(t)}{|r(t)|}. \tag{13}$$

Define a variable

$$y(t) = [r^2(t) - \alpha^2]/\alpha^2, \tag{14}$$

where, α is a variable parameter to regulate the degree of chatter. When

$$|y(t)| \le 1, \quad \text{i.e. } \alpha \ge \frac{\max|r(t)|}{\sqrt{2}}, \tag{15}$$

the absolute value of $r(t)$ can be rewritten in a convergent series expansion expressed by Eq. (16):

$$|r(t)| = \alpha\Big[1 + \frac{1}{2}y(t) - \frac{1}{8}y^2(t) + \frac{1}{16}y^3(t) - \frac{5}{128}y^4(t) + \cdots\Big]. \tag{16}$$

If $|y(t)| > 1$, Eq. (16) will be divergent and the control force will tend to infinity gradually. Integrating the above equations, the control force can be expressed as:

$$u(t) = -u_{\max}r(t)\Big/\Big\{\alpha\Big[1 + \frac{1}{2}y(t) - \frac{1}{8}y^2(t) + \frac{1}{16}y^3(t) - \frac{5}{128}y^4(t) + \cdots\Big]\Big\}. \tag{17}$$

By comparing Eqs. (13) and (17), the technique adopted by the modified Bang–Bang control presented in Ref. [19] can be clearly showed. It uses a series expansion instead of an absolute function. If the parameter α is designed properly, the undesirable control chattering near the origin of the state-space can be eliminated and the high-speed switching problem is solved at the same time. Unfortunately, r and y are coupled with the responses of a structure to earthquake excitations, and the intensity of the ground acceleration is uncertain. Once the condition expressed by Eq. (15) cannot be satisfied, the series will be divergent and the actively controlled system according to Eq. (17) will be unstable.

4 Adaptive Bang–Bang control law

It is paramount to design a Bang–Bang controller that always satisfies the convergent condition given by Eq. (15). Lim et al. [20] used an adaptive algorithm to the modified Bang–Bang control strategy to guarantee the stability in the structural system by changing the value of α. The adaptive modified Bang–Bang control algorithm is expressed as:

$$u(t) = \begin{cases} -u_{\max}r(t)\Big/\Big\{\alpha\Big[1 + \frac{1}{2}y(t) - \frac{1}{8}y^2(t) + \frac{1}{16}y^3(t) - \frac{5}{128}y^4(t) + \cdots\Big]\Big\} & \text{if } \alpha \le \alpha_{\min}, \\ -u_{\max}\mathrm{sgn}(r(t)) & \text{otherwise.} \end{cases} \tag{18}$$

It means that a switch is added to the modified Bang–Bang controller and it is triggered when the necessary value of α to keep the stability of the controller is larger than $\alpha_{\min}$, which is determined in an initial design stage by considering the control performance and control chattering. One can refer to Ref. [20] for more details about the analyses of the adaptive Bang–Bang control algorithm.

5 Improved suboptimal Bang–Bang control law

This paper proposes a series of continuous functions for the suboptimal Bang–Bang control law to improve the nature of this controller. The main technique here is to

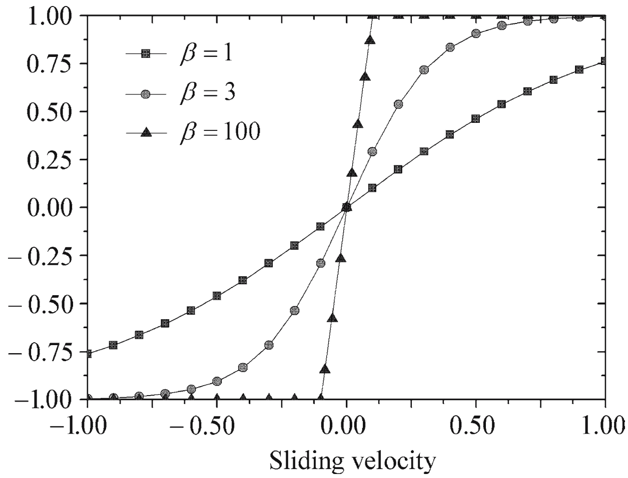

Fig. 1 Plot of hyperbolic tangent function

introduce the continuous functions, as shown by the following four equations [18]: the Tangent Function, Error Function, Arc Tangent Function and Complex Function to approach the signum function at any level of accuracy:

$$
\begin{aligned}
f(\beta, v) &= \tanh(\beta v),\\
f(\beta, v) &= \mathrm{erf}(\beta v),\\
f(\beta, v) &= (2/\pi)\mathrm{Arctan}(\beta v),\\
f(\beta, v) &= \beta v/(1+\beta|v|).
\end{aligned}
\tag{19}
$$

The degree of approximation of the above four functions to the signun function can be adjusted by regulating the value of β. A comparison of these functions with the sliding velocity v and parameter β was given in Ref. [18]. Figure 1 shows the typical plot of a hyperbolic tangent function as the sliding velocities with β equal to 1, 3 and 100.

On the basis of the suboptimal Bang–Bang control approach, the active control force can be modified as:

$$u(t) = -u_{\max}\mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}(t))\tanh[\beta v(t)]. \tag{20}$$

The four previously mentioned problems, i.e. the online-solution of differential equations, high-speed switching, control chattering and instability, can be solved together by a simple expression as Eq. (20). It is clear that the stable suboptimal Bang–Bang control is the main controller and the hyperbolic tangent function acts as a regulator to adjust the control force in terms of the sliding velocity of the damper. Evidently, not only this modified suboptimal control algorithm is unconditionally stable regardless of the intensity of excitation, but also the control force does not switch between the two extreme values drastically and it changes the direction smoothly to avoid chattering at the origin of the state-space. The detailed numerical analyses will be conducted in later sections.

6 Semi-active control strategy for friction dampers

It is a constant challenge to develop a control method for variable friction dampers because of their intrinsically non-linear properties. The control strategies should be developed that are practically implementable and can fully utilize the capabilities of these devices.

The improved suboptimal Bang–Bang control is an actively oriented algorithm. It should be clipped for the control of semi-active friction dampers. Assuming that the sliding velocity of the damper is approximately equal to the inter-story velocity, $\dot{x}_{\mathrm{ins}}$, between the neighbouring floors of a structure where the damper is installed, the clipped control force can be obtained according to the relationship of the sign between $\dot{x}_{\mathrm{ins}}$ and the corresponding value in the vector $\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{Z}(t)$.

The inter-story velocities of a structure can be expressed as:

$$\dot{x}_{\mathrm{ins}}(t) = \boldsymbol{D}\boldsymbol{z}(t), \tag{21}$$

where

$$\boldsymbol{D} = \left[\begin{array}{c|cccccccc} & 1 & -1 & 0 & 0 & 0 & \cdots & 0 & \\ & 0 & 1 & -1 & 0 & 0 & \cdots & 0 & \\ \boldsymbol{0}_{n\times n} & & & & & \ddots & & & \\ & 0 & 0 & 0 & \cdots & 0 & 1 & -1 & \\ & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & \end{array}\right]_{n\times 2n}. \tag{22}$$

When the sign of $\dot{x}_{\mathrm{ins}}$ is the same as that of the corresponding component in the vector $\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{Z}(t)$, the absolute value of the control force is equal to $u_{\max}$. Otherwise, it is equal to zero. Using the original suboptimal Bang–Bang control algorithm, this control strategy can be formulated as follows:

$$f(t) = \begin{cases} -u_{\max}\mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}(t)) - |f_{\mathrm{pre}}|\mathrm{sgn}(\dot{x}_{\mathrm{ins}}(t)), \\ \qquad\qquad \text{if} \quad u_{\mathrm{cal}}\cdot\dot{x}_{\mathrm{ins}}(t) < 0, \\ -|f_{\mathrm{pre}}|\mathrm{sgn}(\dot{x}_{\mathrm{ins}}(t)) \quad \text{otherwise}, \end{cases} \tag{23}$$

$$u_{\mathrm{cal}} = -u_{\max}\mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}(t)). \tag{24}$$

Furthermore, one can write Eq. (23) in a general form represented by Eq. (25) for simplification:

$$
\begin{aligned}
f(t) = -\Big(&\frac{1}{2}u_{\max}\cdot(\boldsymbol{I} + \mathrm{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}(t)\\
&\cdot\boldsymbol{D}\boldsymbol{z}(t))) + f_{\mathrm{pre}}\Big)\cdot\mathrm{sgn}(\dot{x}_{\mathrm{ins}}(t)),
\end{aligned}
\tag{25}
$$

where f_{pre} is the frictional force produced by the preloading normal force N_{pre} acted across the damper; The dots in the above formula represent the dot product.

Using the same control strategy as Eq. (23), another compact formulation can be obtained based on the modified suboptimal Bang–Bang control algorithm formulated by Eq. (20). The control force can be written as:

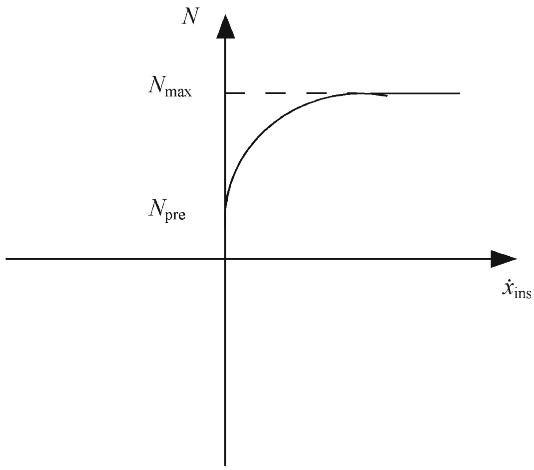

Fig. 2 A sketch of the control strategy proposed in this paper

$$f(t) = -\left(\left(\frac{1}{2}u_{\max} \cdot (\boldsymbol{I} + \text{sgn}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{z}(t) \cdot \boldsymbol{D}\boldsymbol{z}(t)))\right) \cdot g(|\beta \boldsymbol{D}\boldsymbol{z}(t)|) + f_{\text{pre}}\right) \cdot \text{sgn}(\dot{x}_{\text{ins}}(t)), \tag{26}$$

where $g(\cdot)$ indicates suitable continuous functions with characterizations like those given by Eq. (19). The control strategy can intuitively be seen from Fig. 2. The contacting force provided by an actuator smoothly transits from the preloading normal force N_{pre} to the maximal contacting force $N_{\max}$.

Here, we call the above semi-active control strategies for a variable friction damper expressed by Eq. (25) and Eq. (26) as the Bang–Bang-A and Bang–Bang-B control strategies, respectively.

7 Selection of parameters

The selection of parameters in controller is one of the keys in designing a satisfactory control system. The control effectiveness, robustness and other performances of the controller are definitely related to the specially designed parameters. Because of the non-linear nature of Eqs. (25) and (26) it becomes formidable to optimize the parameters included in the damper and the controller that are integrated with structures subjected to ground motion excitations.

As a smart and bionic method, GA is very suitable for optimization of non-linear problems. It is a stochastically global search approach that mimics the natural biological evolution without requiring any derivative information or other auxiliary knowledge of the objective function. GA operates on a population of potential solutions by applying the principle of survival of the fittest to produce hopefully better and better approximations. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness within the problem domain and breeding them together by use of operators borrowed from natural genetics. This course leads to the evolution of populations of individuals that are better suited to their environment than when they were created, just as in the natural adaptation.

To optimize the control force, $u_{\max}$, and the preloading frictional force, f_{pre}, the first step in the algorithm is to code the information about the related quantities into a genetic string by binary alphabets. Each genetic string is chosen to be comprised of n elements, where each element in the string represents a possibly optimal parameter of the damper and controller. Once the information is successfully encoded, genetic computations follow that include selection, cross-over, mutation and other operations to approach the convergent results.

Because of the calibration of individual performance in the problem domain the fitness value of each individual must be determined through optimizing the objective function. In this paper, the relative performance index (RPI) [21] that reflects the structural strain energy is selected as a target, expressed by Eq. (27), to be minimized, considering the relation between the amounts of elastic strain energy imparted into a building and the resulting structural response:

$$\text{RPI} = \frac{1}{2}\left(\frac{\text{SEA}}{\text{SEA}_{(0)}} + \frac{U_{\max}}{U_{\max(0)}}\right), \tag{27}$$

where SEA and $U_{\max}$ are the area under the elastic strain-energy time history and the maximum strain energy for a friction damped structure; $SEA_{(0)}$ and $U_{\max(0)}$ are those of the original uncontrolled structure.

For the evaluation of fitness, its value can be calculated by Eq. (28), so the most optimal individual takes the largest fitness value:

$$\text{Fit} = 1 - \text{RPI}. \tag{28}$$

It is obvious that Fit is a positive value less than 1.

8 Numerical analysis

The intention of the analysis is to obtain a better effectiveness of the improved suboptimal control method for the control of semi-active friction dampers. A linear three-story building model with a semi-active friction damper at the first floor shown in Fig. 3 is taken as the numerical example, in which each story unit is identically constructed. The mass, stiffness and damping coefficients of each story are $m_i = 1{,}000$ kg, $k_i = 980$ kN/m, and $c_i = 1.407$ kN s/m, respectively, for $i = 1, 2$, and 3.

The simulation model is established by using the Simulink® and Stateflow nested in the MATLAB®

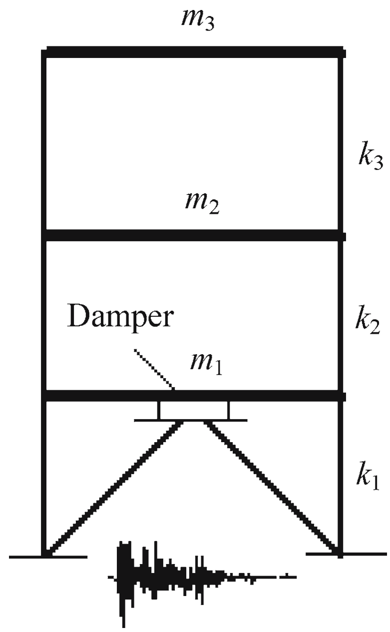

Fig. 3 Three-story building with a semi-active friction damper at the first floor

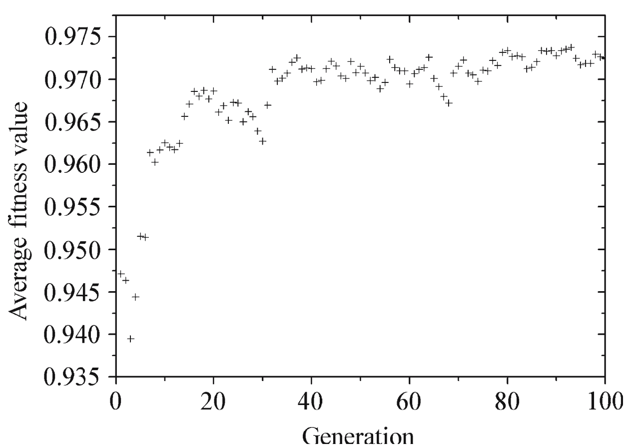

Fig. 4 Average fitness value versus generation

program. The hit crossing block in the Simulink® and the Stateflow operate together to determine in which phase the friction damper is in a state of sliding or sticking. The size of each generation within the GA is chosen to be 100. The evolutional computation is ended until the number of generations exceeds 100. When the structure is excited by the scaled Imperial Valley (May 19, 1940, El Centro, Array #9) earthquake with PGA 0.22 g, the optimized values of $f_{\max}$ and f_{pre} are 4,300 and 540 N, respectively. The average fitness value for each generation is shown in Fig. 4. It can be seen from the figure that the results are convergent at about the 40th generation.

Figures 5 and 6 show the first story's responses between the uncontrolled and controlled structure adopting the Bang–Bang-A and Bang–Bang-B control

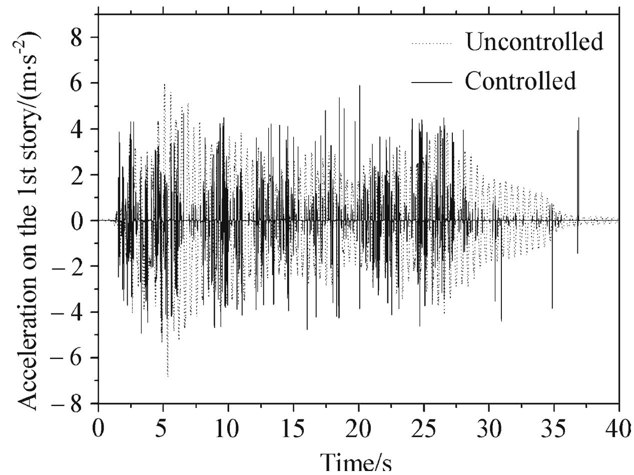

Fig. 5 Bang–Bang-A control strategy

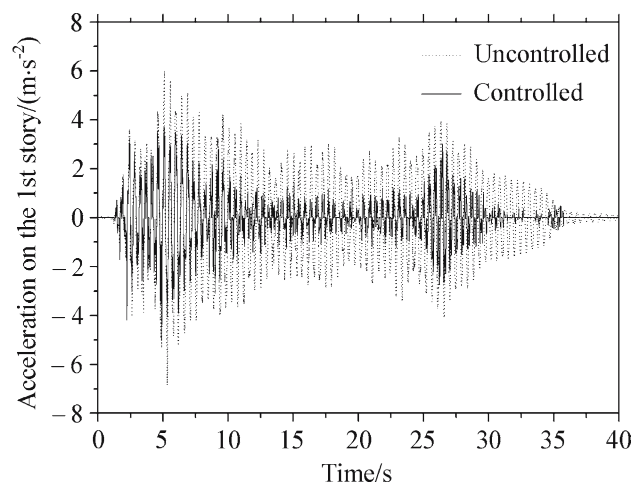

Fig. 6 Bang–Bang-B control strategy

strategies, respectively, when β within the hyperbolic tangent function is equal to 60. The maximum inter-story drifts, relative accelerations and the corresponding reduction ratios (RR) are listed in Table 1.

From the results, especially, those of the first story where the installed, the control effectiveness of the Bang–Bang-B control strategy is control strategy is better than that of the Bang–Bang-A controller. The chattering problem is evident due to the high-speed switching of the normal force when the Bang–Bang-A control strategy is used. After introducing the hyperbolic tangent function, the accelerations on the first story are reduced evidently. The reduction ratio of the first story's acceleration is increased from 1.8 to 27%, and it can be improved further by suitably reducing the value of β. It is a tradeoff between the control effectiveness on accelerations and that on inter-story drifts.

It is worth analyzing the influences of the parameter, β, in the hyperbolic tangent on the controlled responses. Figures 7a and b reflect the control efficiency of the

Table 1 Response quantities

Control strategies		Inter-story drifts/m			Relative accelerations/$(m\cdot s^{-2})$		
		1st	2nd	3rd	1st	2nd	3rd
Bang–Bang-A	Result	0.0037	0.0055	0.0043	5.91	4.18	5.14
	RR	87%	75%	66%	1.8%	56%	59%
Bang–Bang-B	Results	0.0045	0.0056	0.0041	4.38	4.04	5.02
	RR	84%	74%	68%	27%	58%	60%
Without control		0.0286	0.0218	0.0127	6.02	9.56	12.49

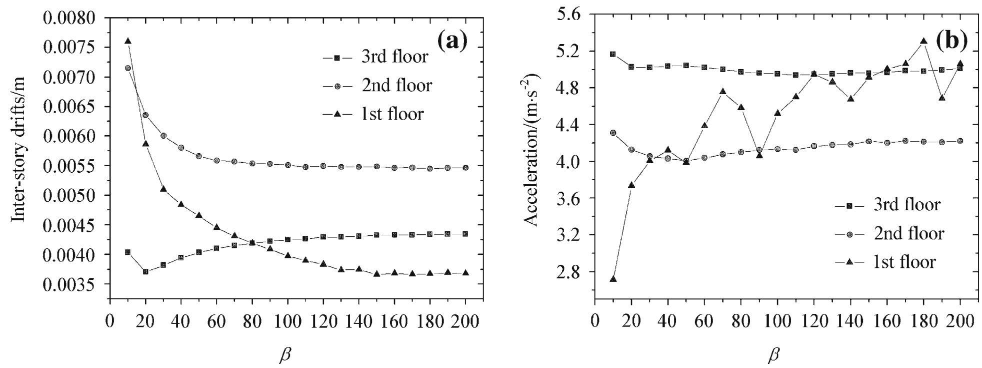

Fig. 7 The influence of β on control efficiency. **a** Inter-story drifts; **b** story accelerations

Bang–Bang-B control strategy as β is increased from 10 to 200.

The effects of β on the structural responses are evident, specially, on the first floor. As β increases, the inter-story drifts of the first floor degrade and the relative accelerations change in the same direction as that of β. Generally, the influences of β on inter-story drifts of the other stories are greater than those on story's accelerations. The chattering problem will be intensified because the action of the hyperbolic tangent function disappears gradually as β increases. In addition, It should be pointed out that another very important factor related to the effect of β is the location of dampers. Different placements of dampers will influence the results of different stories.

Because of the uncertain nature of ground motion accelerations, the adaptability of a controller becomes a point that we must consider for real applications. Using the same parameters in the passive Bang–Bang-A and Bang–Bang-B controls and taking the amplitudes of accelerations as 0.22, 0.1, and 0.035 g, respectively, the results of the reduction ratios of the dissipated energy produced by the friction damper to the total input energy versus the ground motion intensities are shown in Fig. 8, in which the abscissa value 1, 2, and 3 indicate the strong (0.22 g), medium (0.1 g), and small (0.035 g) earthquake motions. It is noted that the Bang–Bang-B strategy has the best flexibility followed by the Bang–Bang-A algorithm. When the structure with the passive energy damper is under the excitation of a small earthquake, the dissipated energy is zero due to its large initial sliding force. Thereby, it can generally be concluded that the semi-active control systems are more effective than the passive one.

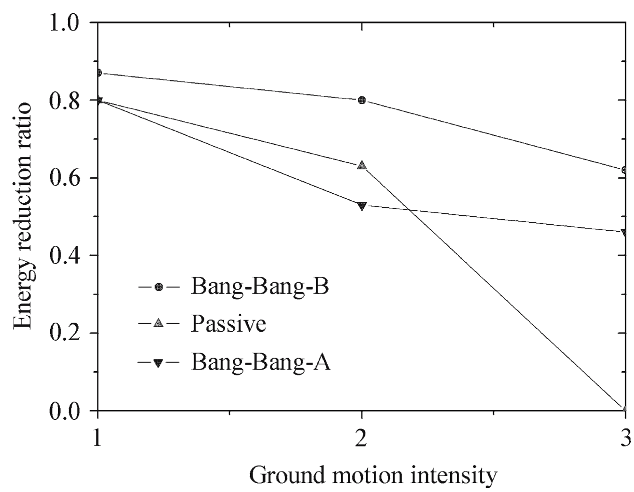

Fig. 8 Comparison of adaptive ability

9 Concluding remarks

This paper presents a new modified method for the improvement of the suboptimal Bang–Bang control algorithm and further develops it to a semi-active control strategy taking into account the properties of friction dampers. The parameters in the system is determined by the GA for obtaining a much better performance of the controller. The approach is simple compared to those utilized by the modified and the adaptive Bang–Bang control systems. In addition, this technique not only can fully avoid the online-calculation of differential equations and solve chattering problems that implicitly exits in the Bang–Bang control algorithm, but also guarantees the stability at any level of earthquake excitations. It has been shown that the modified technique with the control of variable friction dampers can successfully reduce the response of structures during large seismic events. Numerical analyses show that β included in the continuous function obviously influences the control efficiency and the adaptability of the improved controller, which is much more effective than the original one.

Consequently, the merits of the Bang–Bang-B control algorithm are evident. It can make a full use of the capability of an actuator to change the stiffness and damping of the structure. In addition, the implementation of this strategy is also very simple. However, the Bang–Bang-B control method is only suitable for the seismic control of multi-story structures because it needs all states of the structure to determine the magnitude of the control force, and the direction of the force for the active control systems. For high-rise buildings, it may involve a time delay to control the systematic response since a lot of numerical computations need to be performed. That will be our further research work.

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