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均布载荷作用下带边缘大波纹膜片的非线性弯曲

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摘要　采用轴对称旋转壳体的简化 Reissner 方程，研究了在均布载荷作用下具有硬中心的带边缘大波纹膜片的非线性弯曲问题. 应用积分方程方法，获得了具有夹紧固定和滑动固定两种外边界的膜片的特征关系，即荷载 - 中心挠度曲线. 作为算例，给出了夹紧固定膜片中的应力分布.

关键词　波纹膜片，边缘大波纹，非线性弯曲，弹性特征，环形板

引　言

波纹膜片作为精密仪器的一种弹性元件，广泛地应用在测量仪表里. 因此，对于工程界说来，研究这种膜片的非线性弯曲问题十分重要. 使用扁壳的非线性弯曲理论，Panov[1]，Feodosev[2]，陈山林[3]，宋卫平和叶开沅[4]等讨论了浅正弦波纹膜片这种特殊情况. 袁鸿[5]采用摄动法和幂级数方法, 研究了具有光滑中心的锯齿形和梯形波纹壳的弹性特征. 对于波纹膜片分布得均匀致密的膜片, Haringx[6]首先提出将波纹圆板问题转化为等效的正交各向异性板问题的思想，为波纹圆板的理论分析提供了一种新的手段. Andryewa[7,8]，Akasaka[9]等探讨了这一新的思想. 从正交异性板理论出发，刘人怀使用修正迭代法[10~12]成功地解决了一系列波纹圆板和波纹环形板的非线性弯曲问题[13~18].

工程实际中，经常遇到深波纹膜片及带边缘大波纹膜片，这就要求从一般壳体大挠度方程进行求解. 遗憾的是, 由于形状复杂及非线性数学带来的困难, 只有极少数的文献讨论了这一问题. Axelrad[19]讨论了深正弦波纹膜片的伽辽金解法. Hamada 等[20]采用差分方法求解了带边缘波纹膜片的弯曲问题，Bihari 和 Elbert[21]直接从微分方程出发得到了带边缘波纹膜片的特征曲线. 但是他们的结果都不能令人满意，也不能得到膜片中的应力分布. 最近，刘人怀和袁鸿[22]采用格林函数方法，将简化的 Reissner 方程化为积分方程，成功地求解了中心集中载荷作用下带边缘大波纹膜片的弯曲问题，并与其它理论和实验结果做了比较. 本文采用轴对称旋转壳体的简化 Reissner 方程，研究了在均布载荷作用下具有硬中心的带边缘大波纹膜片的非线性特征曲线，给出了膜片中的应力分布. 计算结果可供设计弹性元件时作参考.

2001–06–07 收到第一稿，2002–05–20 收到修改稿.

1 基本方程和边界条件

采用 (r,ϑ,z) 表示右手笛卡儿坐标系 $Oxyz$ 中的圆柱坐标，将一条平面曲线绕 z 轴旋转就得到一轴对称的旋转壳，如图 1 所示.

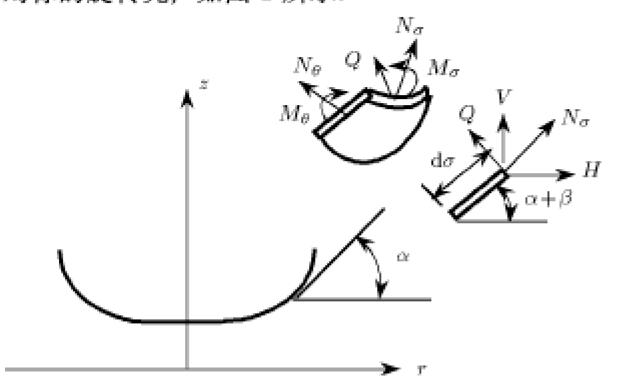

图 1　旋转壳的轴向截面

Fig.1　Geometry of a shell of revolution

轴对称旋转壳体的简化 Reissner 方程为[23]

$$D[(r\beta')'-r^{-1}\sin\beta]-F\sin(\alpha+\beta)+rV\cos(\alpha+\beta)=0 \tag{1}$$

$$A[(rF')' - r^{-1}F] + \cos\alpha - \cos(\alpha+\beta) + A[(r^2P_H)' + \nu rP_T] = 0 \tag{2}$$

上式中

$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad A = \frac{1}{Eh}, \quad (\ \ )' = \frac{\mathrm{d}}{\mathrm{d}\sigma}(\ \ )$$

σ 是子午线方向弧长，在均布载荷 P_0 作用下

$$rV = \frac{1}{2}P_0r^2 \tag{3}$$

$$P_T = 0 \tag{4}$$

$$P_H = P_0\sin(\alpha+\beta) \tag{5}$$

内力及位移与转角 β 及应力函数 F 的关系为

$$H = r^{-1}F \tag{6}$$

$$N_\vartheta = F' + rP_0\sin(\alpha+\beta) \tag{7}$$

$$N_\sigma = r^{-1}F\cos(\alpha+\beta) + \frac{1}{2}P_0r\sin(\alpha+\beta) \tag{8}$$

$$M_\sigma = D\{\beta' + \nu r^{-1}[\sin(\alpha+\beta) - \sin\alpha]\} \tag{9}$$

$$M_\vartheta = D\{r^{-1}[\sin(\alpha+\beta) - \sin\alpha] + \nu\beta'\} \tag{10}$$

$$u = Ar\Big\{F' + rP_0\sin(\alpha+\beta) - \nu\Big[r^{-1}F\cos(\alpha+\beta) + \frac{1}{2}P_0r\sin(\alpha+\beta)\Big]\Big\} \tag{11}$$

$$w = w(0) + \int_0^\sigma [\sin(\alpha+\beta) - \sin\alpha + e_\sigma\sin(\alpha+\beta)]\mathrm{d}\sigma \tag{12}$$

式 (12) 中，e_σ 是子午线方向的薄膜应变

$$e_\sigma = A[N_\sigma - \nu N_\vartheta] \tag{13}$$

考虑图 2 所示带刚性硬中心和边缘大波纹的波纹环形圆板，其外半径为 a，内半径为 b，显然在 $r = b$

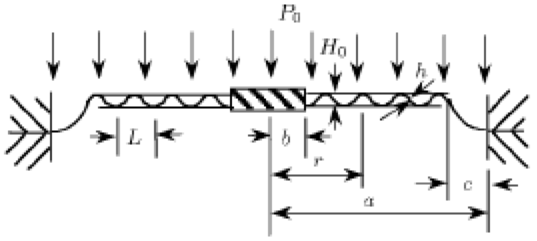

图 2　带边缘大波纹的波纹环形圆板

Fig.2　A corrugated circular annular plate with a large boundary corrugation

处有

$$\beta = 0, \qquad u = 0 \tag{14}$$

讨论两种常用的外边界条件：

1) 夹紧固定情形

$$w = 0,\ \beta = 0,\ u = 0,\ 当\ r = a\ 时 \tag{15}$$

2) 滑动固定情形

$$w = 0,\ \beta = 0,\ H = 0,\ 当\ r = a\ 时 \tag{16}$$

为求解方便，引入无量纲量

$$\left.\begin{aligned} &x = \frac{\sigma}{a}, \quad R = \frac{r}{a}, \quad g = \frac{aF}{D} \\ &\lambda^2 = \frac{AD}{a^2}, \quad P = \frac{P_0a^3}{D}, \quad \rho = \frac{b}{a} \end{aligned}\right\} \tag{17}$$

再引入一个新的自变量

$$\xi = \int_0^x R^{-1}\mathrm{d}x \tag{18}$$

以便将方程 (1)，(2) 中的中括号部分化为常系数算子．考虑到 $\sin\beta$ 的幂级数展开的一次项为 β，同时，也为了得到统一的微分算子，基本方程 (1)，(2) 可以化为

$$\frac{\mathrm{d}^2\beta}{\mathrm{d}\xi^2} - \beta = f_\beta \tag{19}$$

$$\frac{\mathrm{d}^2g}{\mathrm{d}\xi^2} - g = f_g \tag{20}$$

式中

$$f_\beta = gR\sin(\alpha+\beta) - \frac{1}{2}R^3P\cos(\alpha+\beta) + \sin\beta - \beta \tag{21}$$

$$f_g = -\frac{1}{\lambda^2}[\cos\alpha - \cos(\alpha+\beta)]R - P\frac{\mathrm{d}}{\mathrm{d}\xi}[R^2\sin(\alpha+\beta)] \tag{22}$$

边界条件 (14)~(16) 成为

$$\beta = 0,\ \frac{\mathrm{d}g}{\mathrm{d}\xi} - \nu g\cos\alpha = \frac{\nu-2}{2}\rho^2P\sin\alpha,\ 当\ R = \rho \tag{23}$$

夹紧固定情形

$$\left.\begin{aligned} &w = 0, \quad \beta = 0, \\ &\frac{\mathrm{d}g}{\mathrm{d}\xi} - \nu g\cos\alpha = \frac{\nu-2}{2}P\sin\alpha,\ 当\ R = 1\ 时 \end{aligned}\right\} \tag{24}$$

滑动固定情形

$$w = 0, \quad \beta = 0, \quad g = 0, \quad 当\ R = 1\ 时 \tag{25}$$

式 (24),(25) 中的边界条件 $w=0$，只是在求得 β 和 g 后，最后计算挠度式 (12) 时，需要运用到.

2 积分方程及其求解

采用格林函数方法，带刚性硬中心波纹壳在外边界夹紧固定时的基本方程及其边界条件 (19), (20), (23) 和 (24) 可以化为下列积分方程

$$\beta=\int_0^{\xi_N} G(\xi,\eta) f_\beta \mathrm{d}\eta \tag{26}$$

$$g=\int_0^{\xi_N} G_2(\xi,\eta) f_g \mathrm{d}\eta+(C_1\mathrm{e}^{\xi}+C_2\mathrm{e}^{-\xi})P \tag{27}$$

式中 C_1 和 C_2 是常数，由下式确定

$$\begin{aligned}C_1=&\big[(\nu/2-1)\sin\alpha_N(1+\nu\cos\alpha_0)-\\&(\nu/2-1)\rho^2\sin\alpha_0(1+\nu\cos\alpha_N)\mathrm{e}^{-\xi_N}\big]/\\&\big[(1+\nu\cos\alpha_0)(1-\nu\cos\alpha_N)\mathrm{e}^{\xi_N}-\\&(1-\nu\cos\alpha_0)(1+\nu\cos\alpha_N)\mathrm{e}^{-\xi_N}\big]\end{aligned} \tag{28}$$

$$\begin{aligned}C_2=&\big[(\nu/2-1)\sin\alpha_N(1-\nu\cos\alpha_0)-\\&(\nu/2-1)\rho^2\sin\alpha_0(1-\nu\cos\alpha_N)\mathrm{e}^{\xi_N}\big]/\\&\big[(1+\nu\cos\alpha_0)(1-\nu\cos\alpha_N)\mathrm{e}^{\xi_N}-\\&(1-\nu\cos\alpha_0)(1+\nu\cos\alpha_N)\mathrm{e}^{-\xi_N}\big]\end{aligned} \tag{29}$$

ξ_N 是 $R=1$ 处的 ξ 值，α_0 和 α_N 分别表示 $R=\rho$ 和 $R=1$ 处的 α 值，$G(\xi,\eta), G_2(\xi,\eta)$ 是格林函数，根据格林函数的构造理论，从式 (19), 式 (20) 相应的齐次方程的基本解 $\frac{1}{4}(\mathrm{e}^{|\xi-\eta|}-\mathrm{e}^{-|\eta-\xi|})$ 与两个解的叠加 $\frac{1}{4}(\mathrm{e}^{|\xi-\eta|}-\mathrm{e}^{-|\eta-\xi|})+C_1(\eta)\mathrm{e}^{\xi}+C_2(\eta)\mathrm{e}^{-\xi}$ 可以得到格林函数的一般形式，代入相应的齐次边界条件就得到格林函数的具体形式.

$$\begin{aligned}G(\xi,\eta)=&\frac{1}{4}(\mathrm{e}^{|\xi-\eta|}-\mathrm{e}^{-|\eta-\xi|})+\\&\frac{1}{4}\Big\{\big[(\mathrm{e}^{\xi-\xi_N}-\mathrm{e}^{\xi_N-\xi})(\mathrm{e}^{-\eta}-\mathrm{e}^{\eta})+\\&(\mathrm{e}^{\xi_N-\eta}-\mathrm{e}^{\eta-\xi_N})(\mathrm{e}^{\xi}-\mathrm{e}^{-\xi})\big]/(\mathrm{e}^{-\xi_N}-\mathrm{e}^{\xi_N})\Big\}\end{aligned} \tag{30}$$

$$\begin{aligned}\Delta\cdot G_2(\xi,\eta)=&\Delta\cdot\frac{1}{4}(\mathrm{e}^{|\xi-\eta|}-\mathrm{e}^{-|\eta-\xi|})-\\&\frac{1}{2}(1+\nu\cos\alpha_0)(1+\nu\cos\alpha_N)\mathrm{e}^{-\xi_N+\eta+\xi}-\\&\frac{1}{2}(1-\nu\cos\alpha_0)(1-\nu\cos\alpha_N)\mathrm{e}^{\xi_N-\eta-\xi}-\\&\frac{1}{4}[(1-\nu\cos\alpha_0)(1+\nu\cos\alpha_N)\mathrm{e}^{-\xi_N}+\\&(1+\nu\cos\alpha_0)(1-\nu\cos\alpha_N)\mathrm{e}^{\xi_N}](\mathrm{e}^{\eta-\xi}+\mathrm{e}^{\xi-\eta})\end{aligned} \tag{31}$$

式 (31) 中

$$\begin{aligned}\Delta=&(1+\nu\cos\alpha_0)(1-\nu\cos\alpha_N)\mathrm{e}^{\xi_N}-\\&(1+\nu\cos\alpha_N)(1-\nu\cos\alpha_0)\mathrm{e}^{-\xi_N}\end{aligned} \tag{32}$$

同理，带硬中心波纹壳在外边界滑动固定时的积分方程，式 (26), 式 (30) 依然有效，而式 (27)~(29), 式 (31), 式 (32) 由下面各式分别替代

$$g=\int_0^{\xi_N} \bar{G}_2(\xi,\eta) f_g \mathrm{d}\eta+(\bar{C}_1\mathrm{e}^{\xi}+\bar{C}_2\mathrm{e}^{-\xi})P \tag{33}$$

$$\bar{C}_1=\frac{(\nu/2-1)\rho^2\sin\alpha_0\mathrm{e}^{-\xi_N}}{(1-\nu\cos\alpha_0)\mathrm{e}^{-\xi_N}+(1+\nu\cos\alpha_0)\mathrm{e}^{\xi_N}} \tag{34}$$

$$\bar{C}_2=-\bar{C}_1\mathrm{e}^{2\xi_N} \tag{35}$$

$$\begin{aligned}\bar{\Delta}\cdot\bar{G}_2(\xi,\eta)=&\bar{\Delta}\cdot\frac{1}{4}(\mathrm{e}^{|\xi-\eta|}-\mathrm{e}^{-|\eta-\xi|})+\\&\frac{1}{2}(1+\nu\cos\alpha_0)\mathrm{e}^{-\xi_N+\eta+\xi}-\\&\frac{1}{2}(1-\nu\cos\alpha_0)\mathrm{e}^{\xi_N-\eta-\xi}+\\&\frac{1}{4}[(1-\nu\cos\alpha_0)\mathrm{e}^{-\xi_N}-\\&(1+\nu\cos\alpha_0)\mathrm{e}^{\xi_N}](\mathrm{e}^{\eta-\xi}+\mathrm{e}^{\xi-\eta})\end{aligned} \tag{36}$$

式 (36) 中

$$\begin{aligned}\bar{\Delta}=&(1-\nu\cos\alpha_0)\mathrm{e}^{-\xi_N}+\\&(1+\nu\cos\alpha_0)\mathrm{e}^{\xi_N}\end{aligned} \tag{37}$$

由于式 (27) 和式 (33) 中的 f_g 由式 (22) 决定，是 $\frac{\mathrm{d}\beta}{\mathrm{d}\xi}$ 的函数，所以它们实际上是积分微分方程，通过分部积分可以消去微分项. 对于中等转动，β^2 与 1 比较可以忽略不计，式 (26) 和式 (27) 可以化为只含一个未知量 β 的非线性积分方程

$$\beta_\tau=-\frac{1}{\lambda^2}\int_0^{\xi_N}\int_0^{\xi_N}G(\tau,\xi)G_2(\xi,\eta)R_\xi(\sin\alpha_\xi+\beta_\xi\cos\alpha_\xi)\beta_\eta(\sin\alpha_\eta+\\ \frac{1}{2}\beta_\eta\cos\alpha_\eta)R_\eta\mathrm{d}\xi\mathrm{d}\eta+P\int_0^{\xi_N}\int_0^{\xi_N}G(\tau,\xi)G_{2,\eta}(\xi,\eta)R_\xi(\sin\alpha_\xi+\\ \beta_\xi\cos\alpha_\xi)[R_\eta^2(\sin\alpha_\eta+\beta_\eta\cos\alpha_\eta)-R_\xi^2(\sin\alpha_\xi+\beta_\xi\cos\alpha_\xi)]\mathrm{d}\xi\mathrm{d}\eta+\\ P\int_0^{\xi_N}G(\tau,\xi)R_\xi(\sin\alpha_\xi+\beta_\xi\cos\alpha_\xi)\{R_\xi^2(\sin\alpha_\xi+\beta_\xi\cos\alpha_\xi)[G_2(\xi,\xi_N)-G_2(\xi,0)]-\\ G_2(\xi,\xi_N)\sin\alpha_N+G_2(\xi,0)\rho^2\sin\alpha_0+C_1\mathrm{e}^{\xi}+C_2\mathrm{e}^{-\xi}\}\mathrm{d}\xi-\\ \frac{1}{2}P\int_0^{\xi_N}G(\tau,\xi)R_\xi^3(\cos\alpha_\xi-\beta_\xi\sin\alpha_\xi)\mathrm{d}\xi \tag{38}$$

上式中，下标 ξ,η,τ 表示 ξ,η,τ 的函数， $\beta_\tau=\beta(\tau)\cdots$ 等， $G_{2,\eta}(\xi,\eta)=\dfrac{\mathrm{d}}{\mathrm{d}\eta}G_2(\xi,\eta)$ ，积分方程可以按下列格式进行迭代

$$\beta_\tau^{*(m)}=-\frac{1}{\lambda^2}\int_0^{\xi_N}\int_0^{\xi_N}G(\tau,\xi)G_2(\xi,\eta)R_\xi(\sin\alpha_\xi+\beta_\xi^{(m-1)}\cos\alpha_\xi)\cdot\\ \beta_\eta^{*(m)}(\sin\alpha_\eta+\frac{1}{2}\beta_\eta^{(m-1)}\cos\alpha_\eta)R_\eta\mathrm{d}\xi\mathrm{d}\eta+P\int_0^{\xi_N}\int_0^{\xi_N}G(\tau,\xi)G_{2,\eta}(\xi,\eta)\cdot\\ R_\xi(\sin\alpha_\xi+\beta_\xi^{(m-1)}\cos\alpha_\xi)[R_\eta^2(\sin\alpha_\eta+\beta_\eta^{(m-1)}\cos\alpha_\eta)-\\ R_\xi^2(\sin\alpha_\xi+\beta_\xi^{(m-1)}\cos\alpha_\xi)]\mathrm{d}\xi\mathrm{d}\eta+P\int_0^{\xi_N}G(\tau,\xi)R_\xi(\sin\alpha_\xi+\\ \beta_\xi^{(m-1)}\cos\alpha_\xi)\{R_\xi^2(\sin\alpha_\xi+\beta_\xi^{(m-1)}\cos\alpha_\xi)[G_2(\xi,\xi_N)-G_2(\xi,0)]-\\ G_2(\xi,\xi_N)\sin\alpha_N+G_2(\xi,0)\rho^2\sin\alpha_0+C_1\mathrm{e}^{\xi}+C_2\mathrm{e}^{-\xi}\}\mathrm{d}\xi-\\ \frac{1}{2}P\int_0^{\xi_N}G(\tau,\xi)R_\xi^3(\cos\alpha_\xi-\beta_\xi^{(m-1)}\sin\alpha_\xi)\mathrm{d}\xi \tag{39}$$

在上式中 m 是迭代次数, 取 $m=1$, 假定 $\beta^{(0)}=0$, 则第一次迭代求出的 $\beta^{*(1)}$ 就是线性解，为了保证计算结果的收敛性，取

$$\beta^{(m)}=\bar{\bar{\lambda}}\beta^{*(m)}+(1-\bar{\bar{\lambda}})\beta^{(m-1)},\quad 0<\bar{\bar{\lambda}}<1 \tag{40}$$

计算表明，当载荷 P 很小时， $\bar{\bar{\lambda}}$ 取 $0<\bar{\bar{\lambda}}<1$ 中的任何值均能保证迭代的收敛性，取 $\bar{\bar{\lambda}}$ 值接近或等于 1 获得较快的收敛速度，而当载荷 P 较大时， $\bar{\bar{\lambda}}$ 值不能取得过大.

实际计算中，取 $\bar{\bar{\lambda}}=0.8$. 给 P 一个充分小的值，按式 (39), 式 (40) 进行迭代，直到

$$\frac{|\beta^{*(m)}-\beta^{(m-1)}|}{1+|\beta^{*(m)}|}\le 0.0001 \tag{41}$$

这样一来，就得到了对应于这个 P 值的非线性解，以后给 P 以增量, 并把对应于上一个 P 值的收敛解用来开始对新的 P 值实施迭代过程. 以这种方式得到对应于任何载荷值的解答.

式 (39) 中，用 $\overline{G}_2(\xi,\eta)$ 代替 $G_2(\xi,\eta)$, $\overline{C}_1$ 和 $\overline{C}_2$ 代替 C_1 和 C_2 ，就得到相应于外边界滑动固定情形的积分方程迭代格式. 求出 β 后，可以从方程 (27) 得到 g ，从式 (6)～(12) 得到内力及位移值. 根据薄壳理论我们计算得到工程中最重要的膜片上表面和下表面总应力分量

$$\sigma_\sigma\Big(\pm\frac{h}{2}\Big)=\frac{N_\sigma}{h}\mp\frac{6M_\sigma}{h^2} \tag{42}$$

$$\sigma_\vartheta\Big(\pm\frac{h}{2}\Big)=\frac{N_\vartheta}{h}\mp\frac{6M_\vartheta}{h^2} \tag{43}$$

对于位于上下表面之间的任何内部点的总应力值可由上面两式给出的上下表面总应力通过线性插值而得到.

3 算 例

图 2 是带边缘大波纹正弦波纹膜片的轴向截面, 各有关数据为：$E=1.345\times10^5\ \mathrm{N/mm^2},\nu=0.3,$

$h = 0.4\,\text{mm}$, $a = 78\,\text{mm}$, $b = 18\,\text{mm}$, $L = 12\,\text{mm}$, $H_0 = 1.34\,\text{mm}$, $c = 15\,\text{mm}$, 边缘大波纹的曲率半径 $r_0 = 45.32\,\text{mm}$，讨论夹紧固定和滑动固定两种外边界情形. 图 3 绘出了波纹膜片的特征曲线，曲线 1 表示外边界夹紧固定情形，曲线 2 表示外边界滑动固定情形. 从图上可知，当载荷不大时，相应于两种边界条件的特征曲线很接近，随着载荷的增大，与滑动固定情形对应的挠度更大一些.

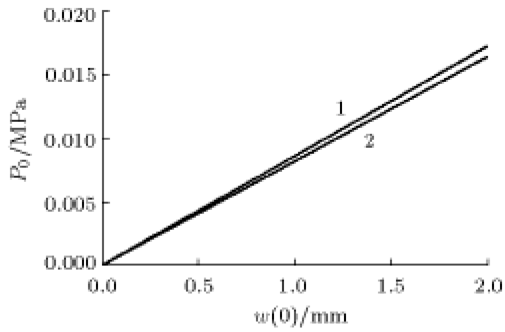

图 3 特征曲线

Fig.3 The characteristic curves

图 4~ 图 7 给出了无量纲均布载荷 $P = 10$ 作用下，外边界夹紧固定时图 2 所示膜片的上下表面环向总应力分布和径向总应力分布. 无量纲总应力 $\sigma_\vartheta^{\rm l}$, $\sigma_\vartheta^{\rm u}$, $\sigma_\sigma^{\rm l}$, $\sigma_\sigma^{\rm u}$ 分别定义为

$$\sigma_\vartheta^{\rm l} = \sigma_\vartheta\Big(-\frac{h}{2}\Big)\frac{12(1-\nu^2)a^2}{Eh^2}$$

$$\sigma_\vartheta^{\rm u} = \sigma_\vartheta\Big(\frac{h}{2}\Big)\frac{12(1-\nu^2)a^2}{Eh^2}$$

$$\sigma_\sigma^{\rm l} = \sigma_\sigma\Big(-\frac{h}{2}\Big)\frac{12(1-\nu^2)a^2}{Eh^2}$$

$$\sigma_\sigma^{\rm u} = \sigma_\sigma\Big(\frac{h}{2}\Big)\frac{12(1-\nu^2)a^2}{Eh^2}$$

由图上可见，上表面和下表面的径向总应力分布 $\sigma_\sigma^{\rm u}$ 和 $\sigma_\sigma^{\rm l}$ 近似反对称于波纹壳的中面，这是由于波纹壳

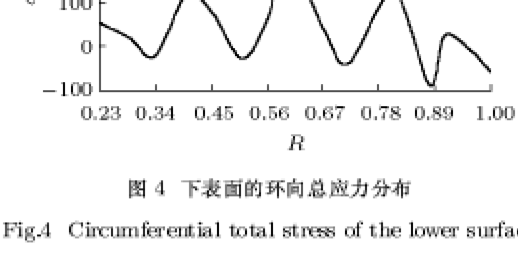

图 4 下表面的环向总应力分布

Fig.4 Circumferential total stress of the lower surface

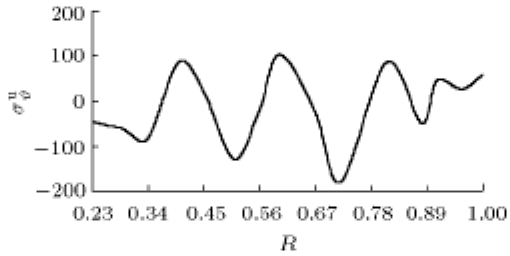

图 5 上表面的环向总应力分布

Fig.5 Circumferential total stress of the upper surface

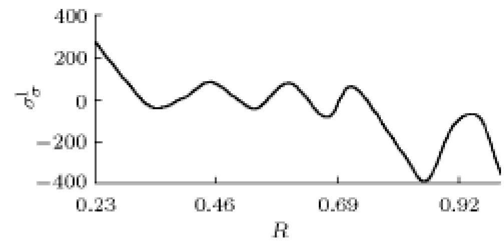

图 6 下表面的径向总应力分布

Fig.6 Meridional total stress of the lower surface

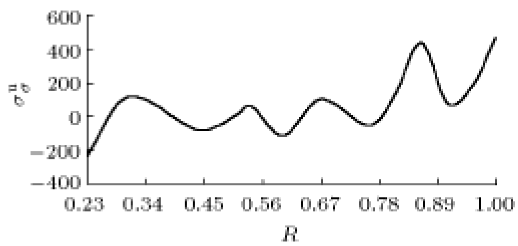

图 7 上表面的径向总应力分布

Fig.7 Meridional total stress of the upper surface

中径向薄膜应力远小于径向弯曲应力的缘故. 由于波纹壳中环向薄膜应力与环向弯曲应力是同等重要的量，因此上表面和下表面的环向总应力 $\sigma_\vartheta^{\rm u}$ 和 $\sigma_\vartheta^{\rm l}$ 不再接近于对称分布. 对于图 2 所示带边缘大波纹膜片，在大波纹的波峰处产生较大应力.

4 结 语

本文从简化的 Reissner 方程出发，采用积分方程方法，得到了在均布载荷作用下具有硬中心和边缘大波纹的波纹壳的非线性弯曲问题的数值解，研究了边界夹紧固定和滑动固定两种情形，得到的特征曲线及壳体内应力分布可供设计参考. 本文中提出的解决方法适应于任意轴向截面的波纹壳体.

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NONLINEAR BENDING OF CORRUGATED DIAPHRAGM WITH LARGE BOUNDARY CORRUGATION UNDER UNIFORM LOAD

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Abstract By using the simplified Reissner's equation of axisymmetric shells of revolution, the nonlinear bending of a corrugated diaphragm with a large boundary corrugation and a non-deformable rigid body at the center under uniform load has been investigated. The nonlinear boundary value problem of the corrugated diaphragm reduces to the nonlinear integral equations by applying the method of Green's function. To solve the integral equations, a so-called interpolated parameter important to the prevention of divergence is introduced into the iterative format. According to the theory of thin shell, the total stress components of upper and lower surface, which are important in engineering, are calculated, after inner forces and displacements are acquired. Both cases of corrugated diaphragm with a rigid clamped edge and a loosely clamped edge are studied. Calculation shows that both characteristic curves of corrugated annular plate with a rigid clamped edge and with a loosely clamped edge are slightly different when load is small. The deflection associated with a loosely clamped edge is larger with increasing the load. Meridional total stress distributions of the upper and lower surface are approximately asymmetric about the middle surface, but circumferential total stress distributions of the upper and lower surface are not approximately asymmetric.

Key words corrugated diaphragm, large boundary corrugation, nonlinear bending, elastic characteristic, annular plate

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