一类网格结构模型的研究 1)

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摘要　提出了一类适用于大规模二维网格结构计算的离散模型. 该模型以网格节点为研究对象，其控制方程仅与跟节点相连的杆件数目及几何形状有关；只要给出几种基本类型的节点控制方程，就可以对具有相应形式节点的各种规模的网格材料组合其控制方程而求解. 这种离散模型形式简单，尤便于在计算机上实现.

关键词　网格结构，离散模型，多胞材料

引　言

网格 (Lattice) 材料属于多胞材料 (Cellular material). 网格材料和其它多胞材料 (如蜂窝材料、泡沫、木材、软木、海绵和骨松质等) 一样具有低的相对密度 ($\rho^*/\rho \leq 0.03$) 、结构周期性、基体材料广泛 (聚合物、金属、陶瓷、玻璃和复合材料等) 、制作工艺简单、比强度高等特点. 因此，这类材料正日益得到广泛的研究和应用，如飞机蒙皮、卫星整流罩、大型体育场馆等.

对多胞材料的研究范围很广，主要包括几何结构，材料性质，力学性质，热学、电学和声学性质，吸能性质等. 近年在该领域内的主要成果总结在 L.J. Gibson 和 M.F. Ashby 的书 [1] 中及相关文献 [2,3]. 我国学者也给出了一些重要结果 [4~7].

文中，我们对结构提出一类离散模型，它适用于所有周期性结构. 在计算多胞材料时一般连续模型的做法是将多胞材料等效为连续介质，优点是计算量小，但精度不够. 而离散模型的特点是毋需等效，可对其进行直接分析，且精度高，但规模庞大，不易实施. 随着计算技术的进步和计算机技术的飞速发展，如计算速度极快的多层网格法 (Multigrid method) ，利用离散模型进行大规模计算正变得可行. 本文所建立的离散模型形式简单，在数据输入和控制方程的组合都非常方便，易于在计算机上实现.

1　二维 Lattice 结构的离散模型

考虑图 1 所示具有 $m \times n$ 个节点的 Lattice 结构，引入坐标系. 对节点 (i,j) ，选取节点位移 $\boldsymbol{u}(i,j) = \{u(i,j), v(i,j), \varphi(i,j)\}^{\mathrm{T}}$ 为基本参量，并规定 u, v 沿坐标轴正向为正， φ 以顺时针转向为正.

1.1　水平杆的刚度方程

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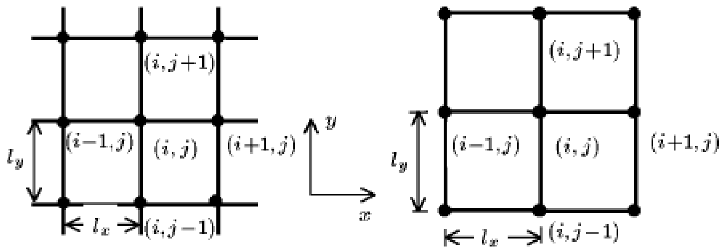


(a) 开放式 (b) 封闭式

(a) Opened lattice (b) Closed lattice

图 1 网格结构

Fig.1 Lattice structure

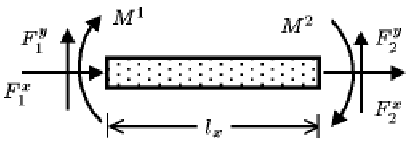


图 2 单杆的内力

Fig.2 Internal forces of a bar

对图 2 所示水平杆, 由 Hooke 定律及 Euler 梁理论 [8,9] ，并假定杆端内力的符号与杆端位移的正向规定一致，则有

$$\boldsymbol{B}^* \boldsymbol{u}^* = \boldsymbol{F}^* \tag{1}$$

其中

$$\boldsymbol{B}^* = \frac{1}{l_x^3}\begin{bmatrix} EAl_x^2 & 0 & 0 & -EAl_x^2 & 0 & 0 \\ 0 & 12EI & -6EIl_x & 0 & -12EI & -6EIl_x \\ 0 & -6EIl_x & 4EIl_x^2 & 0 & 6EIl_x & 2EIl_x^2 \\ -EAl_x^2 & 0 & 0 & EAl_x^2 & 0 & 0 \\ 0 & -12EI & 6EIl_x & 0 & 12EI & 6EIl_x \\ 0 & -6EIl_x & 2EIl_x^2 & 0 & 6EIl_x & 4EIl_x^2 \end{bmatrix} \tag{2}$$

$$\boldsymbol{u}^* = [u^1\ v^1\ \varphi\ u^2\ v^2\ \varphi^2]^{\mathrm{T}}, \quad \boldsymbol{F}^* = [F_1^x\ F_1^y\ M^1\ F_2^x\ F_2^y\ M^2]^{\mathrm{T}} \tag{3}$$

归一化处理，令

$$\boldsymbol{u} = [u_1\ u_2\ u_3\ u_4\ u_5\ u_6]^{\mathrm{T}} = [u^1/l_0\ v^1/l_0\ \varphi^1\ u^2/l_0\ v^2/l_0\ \varphi^2]^{\mathrm{T}} = \boldsymbol{R}\boldsymbol{u}^* \tag{4}$$

$$\boldsymbol{S} = [S_1\ S_2\ S_3\ S_4\ S_5\ S_6]^{\mathrm{T}} = [F_1^x\ F_1^y\ M^1/l_0\ F_2^x\ F_2^y\ M^2/l_0]^{\mathrm{T}}/EA = \boldsymbol{T}\boldsymbol{F}^* \tag{5}$$

又令 $\alpha = 12I/Al_0^2, \beta = l_0/l_x$ ，其中 l_0 是网格结构中最短杆件的长度，而 l_x 是所讨论的杆的长度．则归一化后的刚度方程成为

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{S} \tag{6}$$

其中

$$\boldsymbol{K}=\boldsymbol{T}\boldsymbol{B}^{*}\boldsymbol{R}^{-1}=\frac{\beta}{6}\begin{bmatrix} 6 & 0 & 0 & -6 & 0 & 0 \\ 0 & 6\alpha\beta^2 & -3\alpha\beta & 0 & -6\alpha\beta^2 & -3\alpha\beta \\ 0 & -3\alpha\beta & 2\alpha & 0 & 3\alpha\beta & \alpha \\ -6 & 0 & 0 & 6 & 0 & 0 \\ 0 & -6\alpha\beta^2 & 3\alpha\beta & 0 & 6\alpha\beta^2 & 3\alpha\beta \\ 0 & -3\alpha\beta & \alpha & 0 & 3\alpha\beta & 2\alpha \end{bmatrix} \tag{7}$$

可见刚度矩阵是实对称奇异矩阵，即 $\boldsymbol{K}^{\mathrm{T}}=\boldsymbol{K}, \det\boldsymbol{K}=\boldsymbol{0}$.

1.2 倾斜杆的刚度方程

如图 3 所示倾斜杆，考虑局部坐标系 $o\xi\eta$ 与整体坐标系 oxy 的关系，同样可得到归一化刚度矩阵为

$$\boldsymbol{K}_{\theta}=\boldsymbol{T}\boldsymbol{P}^{-1}\boldsymbol{B}\boldsymbol{P}\boldsymbol{R}^{-1}=\frac{\beta}{6}\times$$

$$\begin{bmatrix} 6(b^2+\alpha\beta^2a^2) & 6(1-\alpha\beta^2)ab & 3\alpha\beta a & -6(b^2+\alpha\beta^2a^2) & 6(-1+\alpha\beta^2)ab & 3\alpha\beta a \\ 6(1-\alpha\beta^2)ab & 6(a^2+\alpha\beta^2b^2) & -3\alpha\beta b & 6(-1+\alpha\beta^2)ab & -6(a^2+\alpha\beta^2b^2) & -3\alpha\beta b \\ 3\alpha\beta a & -3\alpha\beta b & 2\alpha\beta & -3\alpha\beta a & 3\alpha\beta b & \alpha \\ -6(b^2+\alpha\beta^2a^2) & 6(-1+\alpha\beta^2)ab & -3\alpha\beta a & 6(b^2+\alpha\beta^2a^2) & 6(1-\alpha\beta^2)ab & -3\alpha\beta a \\ 6(-1+\alpha\beta^2)ab & -6(a^2-\alpha\beta^2b^2) & 3\alpha\beta b & 6(1-\alpha\beta^2)ab & 6(a^2+\alpha\beta^2b^2) & 3\alpha\beta b \\ 3\alpha\beta a & -3\alpha\beta b & \alpha & -3\alpha\beta a & 3\alpha\beta b & 2\alpha \end{bmatrix} \tag{8}$$

式中 $a=\sin\theta, b=\cos\theta$ ， θ 为倾斜杆与整体坐标系中 x 轴的夹角.

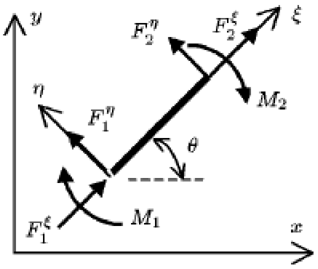


图 3　倾斜杆的杆端力

Fig.3　Bar-end forces of a declining bar

2　十字形网格结构的离散模型

对图 4 所示十字形网格结构，假定各杆的截面性质 (E, I, A 等) 完全相同 (以下同). 考察节点 (i,j) ，将其杆端力自然地取为

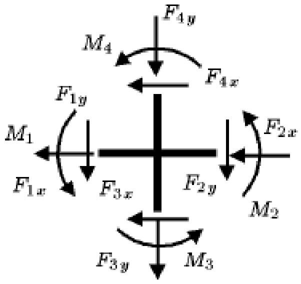


图 4 十字结构

Fig.4 A cross structure

$$\left.\begin{aligned} [F_1^x\ F_1^y\ M_1]^{\mathrm{T}} &= \boldsymbol{G}_1\boldsymbol{K}_0\boldsymbol{u}_{(i,j)(i-1,j)} \\ [F_2^x\ F_2^y\ M_2]^{\mathrm{T}} &= \boldsymbol{G}_2\boldsymbol{K}_0\boldsymbol{u}_{(i,j)(i+1,j)} \\ [F_3^x\ F_3^y\ M_3]^{\mathrm{T}} &= \boldsymbol{G}_1\boldsymbol{K}_{\frac{\pi}{2}}\boldsymbol{u}_{(i,j)(i,j-1)} \\ [F_4^x\ F_4^y\ M_4]^{\mathrm{T}} &= \boldsymbol{G}_2\boldsymbol{K}_{\frac{\pi}{2}}\boldsymbol{u}_{(i,j)(i,j+1)} \end{aligned}\right\} \tag{9}$$

其中 $\boldsymbol{G}_1 = [\boldsymbol{I}_{3\times3}\quad \boldsymbol{0}_{3\times3}]$， $\boldsymbol{G}_2 = [\boldsymbol{0}_{3\times3}\quad \boldsymbol{I}_{3\times3}]$, $\boldsymbol{u}_{(i,j)(i-1,j)}$ 六个分量的顺序有序对 (i,j) 的各指标的升序排列，如对图 4 中节点中的 1 端的位移则表示为节点编号分别为 $(i-1,j)$ 和 (i,j) 的水平杆的杆端位移， 3 端位移则表示为节点编号分别为 $(i,j-1)$ 和 (i,j) 的竖直杆的杆端位移.

由节点的平衡方程得

$$\left.\begin{aligned} & u_{x\bar{x}} + 12I(u_{y\bar{y}} - \varphi_{\bar{y}})/Al_xl_y + F^x_{(i,j)}/EAl_x = 0 \\ & v_{y\bar{y}} + \frac{12I}{Al_xl_y}(v_{x\bar{x}} + \varphi_{\bar{x}}) + \frac{1}{EAl_y}F^y_{(i,j)} = 0 \\ & 4(1/l_x + 1/l_y)\varphi(i,j) + [\varphi(i+1,j) + \varphi(i-1,j)]/l_x + [\varphi(i,j+1) + \varphi(i,j-1)]/l_y + \\ & \qquad 6v_{\bar{x}}/l_x - 6u_{\bar{y}}/l_y = M_{(i,j)}/2EI \end{aligned}\right\} \tag{10}$$

式中

$$\left.\begin{aligned} u_{x\bar{x}} &= (u(i+1,j) - 2u(i,j) + u(i-1,j))/l_x^2 \\ u_{y\bar{y}} &= (u(i,j+1) - 2u(i,j) + u(i,j-1))/l_y^2 \\ v_{x\bar{x}} &= (v(i+1,j) - 2v(i,j) + v(i-1,j))/l_x^2 \\ v_{y\bar{y}} &= (v(i,j+1) - 2v(i,j) + v(i,j-1))/l_y^2 \\ \varphi_{\bar{x}} &= (\varphi(i+1,j) - \varphi(i-1,j))/2l_x \\ \varphi_{\bar{y}} &= (\varphi(i,j+1) - \varphi(i,j-1))/2l_y \\ u_{\bar{x}} &= (u(i+1,j) - u(i-1,j))/2l_x \\ v_{\bar{y}} &= (v(i,j+1) - v(i,j-1))/2l_y \end{aligned}\right\} \tag{11}$$

$u_{x\bar{x}}$ 及 $\varphi_{\bar{x}}$ 等是一种差分格式 (均为中点差分形式). 对式 (10),(11) 使用归一化结果则可改写为 (归一化后的各量加 " * " 号表示)

$$\left.\begin{aligned} & \beta_x^2l_0u^*_{x\bar{x}} + \alpha\beta_x\beta_y(\beta_y^2l_0u^*_{y\bar{y}} - \beta_y\varphi^*_{\bar{y}}) + \beta_xS^{x*}_{(i,j)}/l_0 = 0 \\ & \beta_y^2l_0v^*_{y\bar{y}} + \alpha\beta_x\beta_y(\beta_x^2l_0v^*_{x\bar{x}} + \beta_x\varphi^*_{\bar{x}}) + \beta_yS^{y*}_{(i,j)}/l_0 = 0 \\ & 4(\beta_x + \beta_y)/l_0\varphi^*(i,j) + \beta_x[\varphi^*(i+1,j) + \varphi^*(i-1,j)]/l_0 + \\ & \quad \beta_y[\varphi^*(i,j+1) + \varphi^*(i,j-1)]/l_0 + 6\beta_x^2v^*_{\bar{x}} - 6\beta_y^2u^*_{\bar{y}} = 6\alpha M^*_{(i,j)}/l_0 \end{aligned}\right\} \tag{12}$$

$$\left.\begin{array}{lllll} u_{x\bar{x}}=\beta_x^2 l_0 u_{x\bar{x}}^*\,, & u_{y\bar{y}}=\beta_y^2 l_0 u_{y\bar{y}}^*\,, & v_{x\bar{x}}=\beta_x^2 l_0 v_{x\bar{x}}^*\,, & v_{y\bar{y}}=\beta_y^2 l_0 v_{y\bar{y}}^*\,, & \varphi_{\bar{x}}=\beta_x\varphi_{\bar{x}}^* \\ \varphi_{\bar{y}}=\beta_y\varphi_{\bar{y}}^*\,, & u_{\bar{x}}=\beta_x l_0 u_{\bar{x}}^*\,, & v_{\bar{y}}=\beta_y l_0 v_{\bar{y}}^*\,, & \beta_x=l_0/l_x\,, & \beta_y=l_0/l_y \end{array}\right\} \quad (13)$$

若对式 (13) 部分项级数展开并忽略二阶以上项，并用微分形式表示则为

$$\left.\begin{array}{l} \dfrac{\partial^2 u}{\partial x^2}+\dfrac{12I}{Al_x l_y}\Big(\dfrac{\partial^2 u}{\partial y^2}-\dfrac{\partial\varphi}{\partial y}\Big)+\dfrac{1}{EAl_x}F_{(i,j)}^x=0 \\ \dfrac{\partial^2 v}{\partial y^2}+\dfrac{12I}{Al_x l_y}\Big(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial\varphi}{\partial x}\Big)+\dfrac{1}{EAl_y}F_{(i,j)}^y=0 \\ 6\Big(\dfrac{1}{l_x}+\dfrac{1}{l_y}\Big)\varphi+\dfrac{6}{l_x}\dfrac{\partial v}{\partial x}-\dfrac{6}{l_y}\dfrac{\partial u}{\partial y}=\dfrac{M}{2EI} \end{array}\right\} \quad (14)$$

3 其它重要网格结构模型

网格材料的拓扑结构很多，但由于制作成本和工艺的限制，常见主要有上文给出的十字形结构，此外还有带斜杆的十字结构和蜂窝结构等. 以下，我们给出后两类结构的离散模型.

3.1 带斜杆的十字结构的离散模型

对图 5 所示带一根斜杆的十字结构，同样有

$$\left.\begin{array}{l} u_{x\bar{x}}+12I(u_{y\bar{y}}-\varphi_{\bar{y}})/Al_x l_y+12I\sin\theta\varphi_{\bar{\theta}}/Al_x l+(12I/Al_x l-l/l_x)\sin\theta\cos\theta v_{\theta\bar{\theta}}- \\ \quad (12I\sin^2\theta/Al_x l+l\cos^2\theta/l_x)u_{\theta\bar{\theta}}=F_{(i,j)}^x/EAl_x \\ v_{y\bar{y}}+12I(v_{x\bar{x}}+\varphi_{\bar{x}})/Al_x l_y-12I\cos\theta\varphi_{\bar{\theta}}/Al_y l-(12I/Al_y l-l/l_x)\sin\theta\cos\theta u_{\theta\bar{\theta}}+ \\ \quad (12I\cos^2\theta/Al_{yl}+l\sin^2\theta/l_x)v_{\theta\bar{\theta}}=F_{(i,j)}^y/EAl_y \\ 4(1/l_x+1/l_y+1/l)\varphi(i,j)+[\varphi(i+1,j)+\varphi(i-1,j)]/l_x+[\varphi(i,j+1)+\varphi(i,j-1)]/l_y+ \\ \quad [\varphi(i+1,j+1)+\varphi(i-1,j-1)]/l+6v_{\bar{x}}/l_x-6u_{\bar{y}}/l_y+6\cos\theta v_{\bar{\theta}}/l-6\sin\theta u_{\bar{\theta}}/l= \\ \quad M_{(i,j)}/2EI \end{array}\right\} \quad (15)$$

其中 $l=\sqrt{l_x^2+l_y^2}$ 为斜杆的长度，而

$$u_{\bar{\theta}}=[u(i+1,j+1)-u(i-1,j-1)]/2l$$

$$v_{\bar{\theta}}=[u(i+1,j+1)-u(i-1,j-1)]/2l$$

$$u_{\theta\bar{\theta}}=[u(i+1,j+1)-2u(i,j)+u(i-1,j-1)]/l^2$$

$$v_{\theta\bar{\theta}}=[v(i+1,j+1)-2v(i,j)+v(i-1,j-1)]/l^2$$

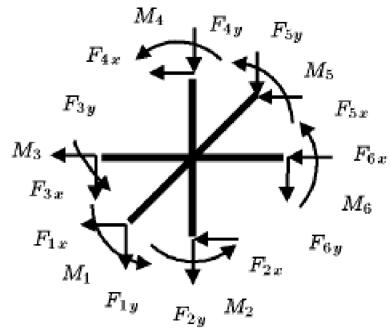


图 5 带一根斜杆的十字结构

Fig.5 Cross structure with a slant bar

容易看出，上述四个量类似于连续函数的一阶和二阶方向导数. 特别是当 $l_x=l_y=l_0$ 时，式 (15) 可改写为

$$
\left.\begin{aligned}
&u_{x\bar{x}}+\alpha(u_{y\bar{y}}-\varphi_{\bar{y}})+\alpha\varphi_{\bar{\theta}}/2+(\alpha/2\sqrt{2}-\sqrt{2}/2)v_{\theta\bar{\theta}}-(\alpha/2\sqrt{2}+\sqrt{2}/2)u_{\theta\bar{\theta}}=F^{x}_{(i,j)}/EAl_x\\
&v_{y\bar{y}}+\alpha(v_{x\bar{x}}+\varphi_{\bar{x}})-\alpha\varphi_{\bar{\theta}}/2-(\alpha/2\sqrt{2}-\sqrt{2}/2)u_{\theta\bar{\theta}}l^2+(\alpha/2\sqrt{2}+\sqrt{2}/2)v_{\theta\bar{\theta}}=F^{y}_{(i,j)}/EAl_y\\
&4(4+\sqrt{2})\varphi(i,j)+[\varphi(i+1,j)+\varphi(i-1,j)]+[\varphi(i,j+1)+\varphi(i,j-1)]+\\
&\qquad[\varphi(i+1,j+1)+\varphi(i-1,j-1)]/\sqrt{2}+6v_{\bar{x}}-6u_{\bar{y}}+3v_{\bar{\theta}}-3u_{\bar{\theta}}=M_{(i,j)}l_0/2EI
\end{aligned}\right\}\tag{16}
$$

式 (16) 归一化结果为

$$
\left.\begin{aligned}
&\beta_x l_0 u^*_{x\bar{x}}+\alpha\beta_y(\beta_y^2 l_0 u^*_{y\bar{y}}-\beta_y\varphi^*_{\bar{y}})+\alpha\beta_\theta^2\sin\theta\varphi^*_{\bar{\theta}}+\beta_\theta^2 l_0(\alpha\beta_\theta-1/\beta_\theta)\sin\theta\cos\theta v^*_{\theta\bar{\theta}}-\\
&\qquad\beta_\theta^2 l_0(\alpha\beta_\theta\sin^2\theta+\cos^2\theta/\beta_\theta)u^*_{\theta\bar{\theta}}=S^{x}_{(i,j)}/l_0\\
&\beta_y l_0 v^*_{y\bar{y}}+\alpha\beta_x(\beta_x^2 l_0 u^*_{x\bar{x}}+\beta_x\varphi^*_{\bar{x}})-\alpha\beta_\theta^2\cos\theta\varphi^*_{\bar{\theta}}-\beta_\theta^2 l_0(\alpha\beta_\theta-1/\beta_\theta)\sin\theta\cos\theta u^*_{\theta\bar{\theta}}+\\
&\qquad\beta_\theta^2 l_0(\alpha\beta_y\beta_\theta\cos^2\theta+\sin^2\theta/\beta_\theta)v^*_{\theta\bar{\theta}}=S^{y}_{(i,j)}/l_0\\
&4(\beta_x+\beta_y+\beta_\theta)\varphi^*(i,j)+\beta_x[\varphi^*(i+1,j)+\varphi^*(i-1,j)]+\beta_y[\varphi^*(i,j+1)+\varphi^*(i,j-1)]+\\
&\qquad\beta_\theta[\varphi^*(i+1,j+1)+\varphi^*(i-1,j-1)]+6\beta_x^2 l_0 v^*_{\bar{x}}-6\beta_y^2 l_0 u^*_{\bar{y}}+6\beta_\theta^2\cos\theta v^*_{\bar{\theta}}-\\
&\qquad6\beta_\theta^2\sin\theta u^*_{\bar{\theta}}=6M^*_{(i,j)}/\alpha
\end{aligned}\right\}\tag{17}
$$

$$
u_{\bar{\theta}}=\beta_\theta u^*_{\bar{\theta}},\quad v_{\bar{\theta}}=\beta_\theta v^*_{\bar{\theta}}v_{\bar{\theta}},\quad \beta_\theta=l_0/l,\quad u_{\theta\bar{\theta}}=\beta_\theta^2 l_0 u^*_{\theta\bar{\theta}},\quad v_{\theta\bar{\theta}}=\beta_\theta^2 l_0 v^*_{\theta\theta}\tag{18}
$$

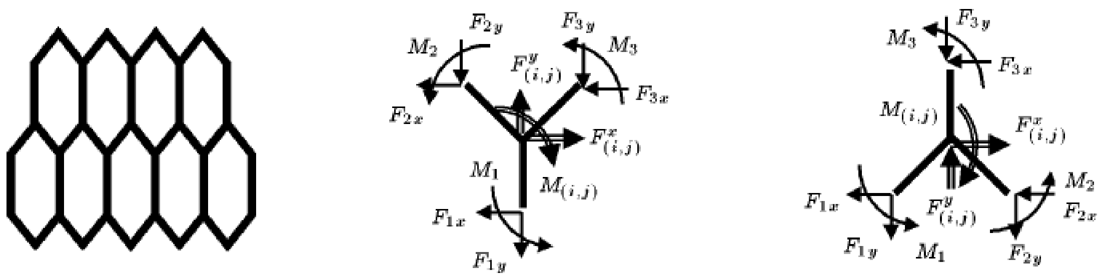
3.2 蜂窝结构的结点模型

蜂窝结构如图 6(a) 所示，对图 6(b) 所示蜂窝结构的“ Y ”字形节点，由其平衡方程可得控制方程为

$$
\left.\begin{aligned}
&\frac{6EI}{l_y^2}[\varphi(i,j)+\varphi(i,j-1)]-\frac{12EI}{l_y^2}u_{\bar{y}}-\Big(EA\cos^2\theta+\frac{12EI}{l^2}\sin^2\theta\Big)(u_{\bar{\theta}}-u_{-\bar{\theta}})-\\
&\quad\Big(EA-\frac{12EI}{l^2}\Big)\sin\theta\cos\theta(v_{\bar{\theta}}+v_{-\bar{\theta}})+\frac{6EI}{l^2}\sin\theta[\varphi(i-1,j)+2\varphi(i,j)+\varphi(i+1,j)]=F^{x}_{(i,j)}\\
&EAu_{\bar{y}}-\Big(EA-\frac{12EI}{l^2}\Big)\sin\theta\cos\theta(u_{\bar{\theta}}+u_{-\bar{\theta}})+\Big(EA\sin^2\theta+\frac{12EI}{l^2}\cos^2\theta\Big)(-v_{\bar{\theta}}+v_{-\bar{\theta}})+\\
&\quad\frac{6EI}{l^2}\cos\theta[\varphi(i-1,j)-\varphi(i+1,j)]=F^{y}_{(i,j)}\\
&4EI\Big(\frac{1}{l_y}+\frac{2}{l}\Big)\varphi(i,j)+2EI\Big(\frac{\varphi(i,j-1)}{l_y}+\frac{\varphi(i+1,j)}{l}+\frac{\varphi(i-1,j)}{l}\Big)-\frac{6EI}{l_y}u_{\bar{y}}+\\
&\quad\frac{6EI}{l}\sin\theta(-u_{\bar{\theta}}+u_{-\bar{\theta}})+\frac{6EI}{l}\cos\theta(v_{\bar{\theta}}+v_{-\bar{\theta}})=M_{(i,j)}
\end{aligned}\right\}\tag{19}
$$

其中 $u_{-\bar{\theta}}=[u(i,j)-u(i-1,j)]/l$, $u_{\bar{\theta}}=(u(i+1,j)-u(i,j))/l$ 等类似于方向导数. 对于正六边形，即 $l=l_y$ ， $\theta=30^\circ$ ，则式 (16) 可改写为

$$\left.\begin{aligned}
&6Il[\varphi(i,j)+\varphi(i,j-1)]-12Ilu_{\bar{y}}-\frac{1}{4}(3Al^2+12I)(u_{\bar{\theta}}-u_{-\bar{\theta}})-\\
&\quad\frac{\sqrt{3}}{4}(Al^2-12I)(v_{\bar{\theta}}+v_{-\bar{\theta}})+3I[\varphi(i-1,j)+2\varphi(i,j)+\varphi(i+1,j)]=\frac{l^2}{E}F^x_{(i,j)}\\
&Al^2u_{\bar{y}}-\frac{\sqrt{3}}{4}(Al^2-12I)(u_{\bar{\theta}}+u_{-\bar{\theta}})+\frac{1}{4}(Al^2+36I)(-v_{\bar{\theta}}+v_{-\bar{\theta}})+\\
&\quad3\sqrt{3}I[\varphi(i-1,j)-\varphi(i+1,j)]=\frac{l^2}{E}F^y_{(i,j)}\\
&12\varphi(i,j)+2(\varphi(i,j-1)+\varphi(i+1,j)+\varphi(i-1,j))-6u_{\bar{y}}+3(-u_{\bar{\theta}}+u_{-\bar{\theta}})+\\
&\quad3\sqrt{3}(v_{\bar{\theta}}+v_{-\bar{\theta}})=\frac{l}{EI}M_{(i,j)}
\end{aligned}\right\}\tag{20}$$



(a) 蜂窝结构　(a) Honeycomb cells

(b) "Y" 字形节点　(b) A Y node

(c) 倒 "Y" 字形节点　(c) A inverse Y node

图 6

Fig.6

而对于图 6(c) 所示倒 " Y " 字形节点，同样可以得到类似的刚度方程如下

$$\left.\begin{aligned}
&-6EI[\varphi(i,j)+\varphi(i,j+1)]/l_y^2+12EIu_{\bar{y}}/l_y^2+(EA\cos^2\theta+12EI\sin^2\theta/l^2)(u_{\bar{\theta}}-u_{-\bar{\theta}})+\\
&\quad(EA-12EI/l^2)\sin\theta\cos\theta(v_{\bar{\theta}}+v_{-\bar{\theta}})-6EI\sin\theta[\varphi(i-1,j)+\\
&\quad2\varphi(i,j)+\varphi(i+1,j)]/l^2=F^x_{(i,j)}\\
&-EAv_{\bar{y}}+(EA-12EI/l^2)\sin\theta\cos\theta(u_{\bar{\theta}}+u_{-\bar{\theta}})+(EA\sin^2\theta+12EI\cos^2\theta/l^2)(v_{\bar{\theta}}-v_{-\bar{\theta}})+\\
&\quad\frac{6EI}{l^2}\cos\theta[\varphi(i-1,j)-\varphi(i+1,j)]=F^y_{(i,j)}\\
&4EI(1/l_y+2/l)\varphi(i,j)+2EI(\varphi(i,j+1)/l_y+\varphi(i-1,j)/l+\varphi(i+1,j)/l)-\\
&\quad6EIu_{\bar{y}}/l_y+6EI\sin\theta(-u_{\bar{\theta}}+u_{-\bar{\theta}})/l+6EI\cos\theta(v_{\bar{\theta}}+v_{-\bar{\theta}})/l=M_{(i,j)}
\end{aligned}\right\}\tag{21}$$

类似于 (17) 式，则有

$$\left.\begin{aligned}&-6I[\varphi(i,j)+\varphi(i,j+1)]+12Iu_{\bar y}+\frac{1}{4}(3EAl^2+12I)(u_{\bar\theta}-u_{-\bar\theta})+\\&\quad\frac{\sqrt3}{4}(Al^2-12I)(v_{\bar\theta}+v_{-\bar\theta})-3I[\varphi(i-1,j)+2\varphi(i,j)+\varphi(i+1,j)]=\frac{l^2}{E}F^x_{(i,j)}\\&-Al^2v_{\bar y}+\frac{\sqrt3}{4}(Al^2-12I)(u_{\bar\theta}+u_{-\bar\theta})+\frac{1}{4}(Al^2+36I)(v_{\bar\theta}-v_{-\bar\theta})+\\&\quad3\sqrt3I[\varphi(i-1,j)-\varphi(i+1,j)]=\frac{l^2}{E}F^y_{(i,j)}\\&12\varphi(i,j)+2(\varphi(i,j+1)+\varphi(i-1,j)+\varphi(i+1,j))-6u_{\bar y}+3(-u_{\bar\theta}+u_{-\bar\theta})+\\&\quad3\sqrt3(v_{\bar\theta}+v_{-\bar\theta})=\frac{l}{EI}M_{(i,j)}\end{aligned}\right\}\tag{22}$$

式 (20), 式 (21) 归一化后分别成为

$$\left.\begin{aligned}&\frac12\alpha\beta_y^2[\varphi^*(i,j)+\varphi^*(i,j-1)]-\alpha\beta_y^3l_0u^*_{\bar y}-\beta_\theta(\cos^2\theta+\alpha\beta_\theta^2\sin^2\theta)(u^*_{\bar\theta}-u^*_{-\bar\theta})-\\&\quad(1-\alpha\beta_\theta^2)\beta_\theta\sin\theta\cos\theta(v^*_{\bar\theta}+v^*_{-\bar\theta})+\frac12\alpha\beta_\theta\sin\theta[\varphi^*(i-1,j)+\\&\quad2\varphi^*(i,j)+\varphi^*(i+1,j)]=S^{x*}_{(i,j)}\\&\beta_yl_0u^*_{\bar y}-(1-\alpha\beta_\theta^2)\beta\sin\theta\cos\theta(u^*_{\bar\theta}+u^*_{-\bar\theta})+\beta_\theta(\sin^2\theta+\alpha\beta_\theta^2\cos^2\theta)(-v^*_{\bar\theta}+v^*_{-\bar\theta})+\\&\quad\frac12\alpha\beta_\theta l_0\cos\theta[\varphi^*(i-1,j)-\varphi^*(i+1,j)]=S^{y*}_{(i,j)}\\&4(\beta_y+2\beta_\theta)\varphi^*(i,j)+2(\beta_y\varphi^*(i,j-1)+\beta_\theta\varphi^*(i+1,j)+\beta_\theta\varphi^*(i-1,j))-\\&\quad6\beta_y^2l_0u^*_{\bar y}+6\beta_\theta^2\sin\theta(-u^*_{\bar\theta}+u^*_{-\bar\theta})+6\beta_\theta^2\cos\theta(v^*_{\bar\theta}+v^*_{-\bar\theta})=\frac{M^*_{(i,j)}}{12\alpha}\end{aligned}\right\}\tag{23}$$

$$\left.\begin{aligned}&-\frac12\alpha\beta_y^2[\varphi^*(i,j)+\varphi^*(i,j+1)]+\alpha\beta_y^3l_0u^*_{\bar y}+\beta_\theta(\cos^2\theta+\alpha\beta_\theta^2\sin^2\theta)(u^*_{\bar\theta}-u^*_{-\bar\theta})+\\&\quad(1-\alpha\beta_\theta^2)\beta_\theta\sin\theta\cos\theta(v^*_{\bar\theta}+v^*_{-\bar\theta})-\frac12\alpha\beta_\theta^2\sin\theta[\varphi^*(i-1,j)+2\varphi^*(i,j)+\varphi^*(i+1,j)]=S^{x*}_{(i,j)}\\&-\beta_yl_0v^*_{\bar y}+(1-\alpha\beta_\theta^2)\beta_\theta\sin\theta\cos\theta(u^*_{\bar\theta}+u^*_{-\bar\theta})+\beta_\theta(\sin^2\theta+\alpha\beta_\theta^2\cos^2\theta)(v^*_{\bar\theta}-v^*_{-\bar\theta})+\\&\quad\frac12\alpha\beta_\theta^2\cos\theta[\varphi^*(i-1,j)-\varphi^*(i+1,j)]=S^{y*}_{(i,j)}\\&2(\beta_y+2\beta_\theta)\varphi^*(i,j)+(\beta_y\varphi^*(i,j+1)+\beta_\theta\varphi^*(i-1,j)+\beta_\theta\varphi^*(i+1,j))-\\&\quad3\beta_y^2l_0u^*_{\bar y}+3\beta_\theta^2\sin\theta(-u^*_{\bar\theta}+u^*_{-\bar\theta})+3\beta_\theta^2\cos\theta(v^*_{\bar\theta}+v^*_{-\bar\theta})=\frac{M^*_{(i,j)}}{6\alpha}\end{aligned}\right\}\tag{24}$$

4 结果与讨论

本文所建立的离散模型形式简单，便于对大规模网格结构进行计算. 其优点在于：

(1) 该模型只需计算几种典型网格结构的离散模型，即根据与节点连接的杆件数目及几何

形状建立控制方程，然后扩充得到整体刚度方程 (或刚度矩阵). 适合于编制程序对任意规模的周期性网格结构进行计算.

(2) 节点坐标的引入，便于原始数据的输入. 将节点位置用坐标标出，不仅对各种规模的网格结构有效，而且可将各杆的特征参量用其端点的坐标变化规律周期性地表出.

(3) 归一化处理使得具体计算时只需考虑杆件参数 α 和 β，而后通过各杆两端节点坐标的规律就可确定其值.

(4) 式 (11)，式 (17)，式 (23) 和式 (24) 事实上是一种差分形式，它们均可近似表为式 (14) 一样的连续模型形式，这对检验该离散模型的可靠性和其计算结果的准确性提供了非常有效的理论基础.

本模型使用了 Euler 梁理论，这对于较短粗的杆是有一定误差的. 但至今为止，尚未见更好的短梁模型，惟有进行数值计算方可. 可见本模型一定程度上适用于较稀疏的网格结构.

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ON A DISCRETE MODEL OF LATTICE STRUCTURE[1)]

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Abstract　In this paper a class of discrete models are proposed which are suitable to evaluate a variety of two-dimension lattice structures. The research object of the model is the node of lattice, and its governing equations are related to the number of bars connecting with the node and the geometry. If only some types of governing equations of essential nodes are presented, the evaluation of cellular solids can be processed for every type of lattice with a corresponding type node. The form of this model is simple and easy to carry out by computer.

Key words　lattice structure, disperse model, cellular solid

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