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饱和地基上含刚核弹性圆板的竖向振动分析 1)

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摘要 基于作者提出的饱和土弹性波动方程, 研究了饱和地基上含刚核弹性圆板在竖向集中谐和力作用下的振动特性. 首先应用 Hankel 积分变换求解该饱和土波动方程, 然后按混合边值条件建立起一组描述含刚核弹性圆板振动的对偶积分方程, 并将其化为第二类 Fredholm 积分方程进行数值求解. 文末给出了含刚核弹性圆板在饱和地基上振动的阻抗函数随无量纲频率 a_0 的变化曲线, 并考察了土的渗透系数, 弹性板含刚域的大小以及板的柔度等参数对阻抗函数的影响, 得出了一些有意义的结论.

关键词 饱和地基, 振动, 含刚核弹性圆板, 阻抗函数, 第二类 Fredholm 积分方程, Hankel 变换

引 言

饱和地基上基础振动动力 - 位移关系的确定是基础半空间理论及土和结构相互作用问题的基本课题. 1971 年 Luco 等 [1] 总结了单相弹性介质半空间体上刚性基础的竖向振动、扭转振动、摇摆振动及水平振动, 并给出了相应的动力柔度系数曲线. Lin[2] 1978 年曾研究了单相黏弹性介质上弹性圆板的竖向振动问题. Iguchi 等 [3] 则研究了带刚域弹性板在单相黏弹性介质上的动力响应.

饱和地基中因存在孔隙水, 且其和土骨架之间可发生相对运动, 故宜将其视作两相介质来研究. 1989 年 Philippacopoulos[4] 以 Biot 理论为基础研究了上层为单相介质弹性固体, 下层为多孔饱和介质的地基上刚体的垂直振动问题. 最近文献 [5, 6] 亦探讨了饱和地基上刚性基础与弹性圆板的垂直振动问题. 但实际工程中, 由于基础板的非均匀性, 因此视基础为刚性板或均匀弹性板都不能很好地反映实际情况, 从已有建筑物的现场动力测试结果来看也确实如此 [7]. 本文为更好地模拟实际工程, 假定基础为一含刚核的弹性板, 并采用作者曾提出的一组弹性土波动方程 [8], 在此基础上求解饱和地基上含刚核弹性板的竖向振动混合边值问题, 文末给出了若干算例, 并考察了板的柔度, 刚域的相对大小及土的渗透性对阻抗函数的影响, 得出了一些对工程实验有意义的结论.

1 基本动力方程及其通解

根据文献 [5], 在圆柱坐标系下饱和土的无量纲波动方程及物理方程可表示为

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$$\left(\overline{\lambda}+1+\frac{\overline{E}_w}{n}\right)\frac{\partial e}{\partial \overline{r}}+\left(\nabla^2-\frac{1}{\overline{r}^2}\right)\overline{w}_r+\frac{\overline{E}_w}{n}\frac{\partial \varepsilon}{\partial \overline{r}}=-\left(\overline{\rho}_w a_0^2\overline{v}_r+a_0^2\overline{w}_r\right) \tag{1}$$

$$\left(\overline{\lambda}+1+\frac{\overline{E}_w}{n}\right)\frac{\partial e}{\partial \overline{z}}+\nabla^2\overline{w}_z+\frac{\overline{E}_w}{n}\frac{\partial \varepsilon}{\partial \overline{z}}=-\left(\overline{\rho}_w a_0^2\overline{v}_z+a_0^2\overline{w}_z\right) \tag{2}$$

$$\frac{\overline{E}_w}{n}\frac{\partial e}{\partial \overline{r}}+\frac{\overline{E}_w}{n}\frac{\partial \varepsilon}{\partial \overline{r}}=-\overline{\rho}_1 a_0^2\overline{w}_r-\overline{\rho}_w a_0^2\overline{v}_r+\mathrm{i}\overline{b}a_0\frac{\overline{v}_r}{n} \tag{3}$$

$$\frac{\overline{E}_w}{n}\frac{\partial e}{\partial \overline{z}}+\frac{\overline{E}_w}{n}\frac{\partial \varepsilon}{\partial \overline{z}}=-\overline{\rho}_1 a_0^2\overline{w}_z-\overline{\rho}_w a_0^2\overline{v}_z+\mathrm{i}\overline{b}a_0\frac{\overline{v}_z}{n} \tag{4}$$

$$\overline{\sigma}_{zw}=2\frac{\partial \overline{w}_z}{\partial \overline{z}}+\overline{\lambda}\left(\frac{\partial \overline{w}_r}{\partial \overline{r}}+\frac{\overline{w}_r}{\overline{r}}+\frac{\partial \overline{w}_z}{\partial \overline{z}}\right) \tag{5}$$

$$\overline{\tau}_{zrw}=\frac{\partial \overline{w}_r}{\partial \overline{z}}+\frac{\partial \overline{w}_z}{\partial \overline{r}} \tag{6}$$

$$\overline{p}_w=\frac{\overline{E}_w}{n}\left(\frac{\partial \overline{v}_r}{\partial \overline{r}}+\frac{\overline{v}_r}{\overline{r}}+\frac{\partial \overline{v}_z}{\partial \overline{z}}+\frac{\partial \overline{w}_r}{\partial \overline{r}}+\frac{\overline{w}_r}{\overline{r}}+\frac{\partial \overline{w}_z}{\partial \overline{z}}\right) \tag{7}$$

式中各变量及参数的意义同文献 [5].

通过 Hankel 积分变换技术，对式 (1)~(7) 进行求解，我们可得各位移分量及应力分量在 Hankel 变换域的解 [5]

$$\widetilde{\overline{w}}_z^0(p,\overline{z})=-c\widetilde{A}_1e^{-c\overline{z}}-d\widetilde{A}_2e^{-d\overline{z}}+p^2\widetilde{A}_3e^{-j\overline{z}} \tag{8}$$

$$\widetilde{\overline{w}}_r^1(p,\overline{z})=-p\widetilde{A}_1e^{-c\overline{z}}-p\widetilde{A}_2e^{-d\overline{z}}+pj\widetilde{A}_3e^{-j\overline{z}} \tag{9}$$

$$\widetilde{\overline{v}}_z^0(p,\overline{z})=-\delta_1c\widetilde{A}_1e^{-c\overline{z}}-\delta_2d\widetilde{A}_2e^{-d\overline{z}}+\delta_3p^2\widetilde{A}_3e^{-j\overline{z}} \tag{10}$$

$$\widetilde{\overline{v}}_r^1(p,\overline{z})=-p\left(\delta_1\widetilde{A}_1e^{-c\overline{z}}+\delta_2\widetilde{A}_2e^{-d\overline{z}}-\delta_3j\widetilde{A}_3e^{-j\overline{z}}\right) \tag{11}$$

$$\widetilde{\overline{\sigma}}_{zw}^0(p,\overline{z})=(\overline{M}c^2-\overline{\lambda}p^2)\widetilde{A}_1e^{-c\overline{z}}+(\overline{M}d^2-\overline{\lambda}p^2)\widetilde{A}_2e^{-d\overline{z}}-2p^2j\widetilde{A}_3e^{-j\overline{z}} \tag{12}$$

$$\widetilde{\overline{\tau}}_{zrw}^1(p,\overline{z})=2pc\widetilde{A}_1e^{-c\overline{z}}+2pd\widetilde{A}_2e^{-d\overline{z}}-p(p^2+j^2)\widetilde{A}_3e^{-j\overline{z}} \tag{13}$$

$$\widetilde{\overline{p}}_w^0(p,\overline{z})=\frac{\overline{E}_w}{n}(c^2-p^2)(1+\delta_1)\widetilde{A}_1e^{-c\overline{z}}+\frac{\overline{E}_w}{n}(d^2-p^2)(1+\delta_2)\widetilde{A}_2e^{-d\overline{z}} \tag{14}$$

这里，$\overline{M}=\overline{\lambda}+2$; $\widetilde{\overline{w}}_r^1,\widetilde{\overline{w}}_z^0,\widetilde{\overline{v}}_r^1,\widetilde{\overline{v}}_z^0$ 分别为土骨架及水相的径向、竖向位移分量的 Hankel 变换式，$\widetilde{\overline{\sigma}}_{zw}^0,\widetilde{\overline{\tau}}_{zrw}^1,\widetilde{\overline{p}}_w^0$ 分别为土骨架的正应力、剪应力及孔隙水压力. $\widetilde{A}_1,\widetilde{A}_2,\widetilde{A}_3$ 是 p 的任意函数，$c=\sqrt{p^2-p_1^2}$, $d=\sqrt{p^2-p_2^2}$, $j=\sqrt{p^2-s^2}$, 而 p_1,p_2,s 分别为对应第一 P 波、第二 P 波和 S 波的无量纲复波数. $\delta_j=-\frac{n}{\mathrm{i}\overline{b}a_0}\left(\overline{\rho}_2a_0^2-\overline{M}p_j^2\right)$, $j=1,2$; $\delta_3=-\frac{n}{\mathrm{i}\overline{b}a_0}\left(\overline{\rho}_2a_0^2-s^2\right)$. 为确定表达式中的 $\widetilde{A}_1,\widetilde{A}_2,\widetilde{A}_3$, 我们必须考虑边界条件.

2 混合边值问题

基础振动的力学模型如图 1 所示，半径为 r_0 的圆板支承在饱和地基半空间表面，而其刚域半径为 r_b, 圆板的泊松比为 ν_f, 挠曲刚度为 D, 在刚核中心承受竖向简谐激振力 $P\cdot e^{\mathrm{i}\omega t}$, ω 为振动圆频率.

假设圆板与饱和土体为光滑接触，且饱和地基表面是透水的，则无量纲化后的地基表面边界条件为

$$\overline{\tau}_{zrw}(\overline{r},0)=0 \qquad (0\le\overline{r}\le\infty) \tag{15a}$$

$$\overline{\sigma}_{zw}(\overline{r},0)=0 \qquad (1\le\overline{r}\le\infty) \tag{15b}$$

$$\overline{p}_{w}(\overline{r},0)=0 \qquad (0\le\overline{r}\le\infty) \tag{15c}$$

$$\overline{w}_{z}(\overline{r},0)=\overline{\Delta}_v-H(\overline{r}-\overline{r}_b)\overline{w}(\overline{r}) \qquad (0\le\overline{r}\le 1) \tag{15d}$$

式中 $\overline{\Delta}_v=\dfrac{\Delta_v}{r_0}$, $\overline{w}=\dfrac{w}{r_0}$, 而 Δ_v 为圆板刚核的竖向位移，w 为圆板相对于刚核的挠曲．$H(x)$ 是 Heaviside 函数．

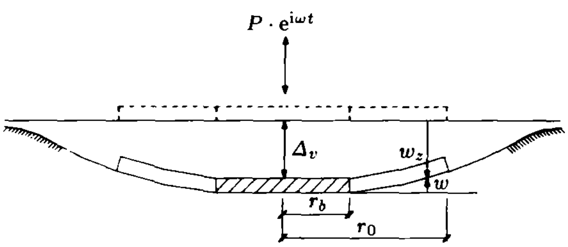

图 1 含刚核弹性圆板振动的力学模型

Fig.1 Geometry of the vibration of an elastic circular plate with rigid core

根据式 (12)~(14) 和式 (15) 我们可求得 $\widetilde{A}_1,\widetilde{A}_2,\widetilde{A}_3$, 然后反代入式 (8), 则有

$$\overline{w}_z(\overline{r},0)=\int_0^\infty p\widetilde{\overline{w}}_z^0(p,0)\mathrm{J}_0(p\overline{r})\mathrm{d}p=\int_0^\infty p\cdot f(p)\widetilde{\overline{\sigma}}_{zw}^0(p,0)\mathrm{J}_0(p\overline{r})\mathrm{d}p \tag{16}$$

式中 $f(p)=\dfrac{s^2(da_1-ca_2)}{(2p^2-\overline{M}p_2^2)(2p_2-s^2)a_1-(2p^2-\overline{M}p_1^2)(2p_2-s^2)a_2-4p^2j(a_1d-a_2c)}$, 而 $a_1=(1+\delta_1)p_1^2$, $a_2=(1+\delta_2)p_2^2$.

若记 $B(p)=p\cdot\widetilde{\overline{\sigma}}_{zw}^0(p,0)$, 将式 (15d) 代入式 (16) 有

$$\int_0^\infty p^{-1}(1+H(p))B(p)\mathrm{J}_0(p\overline{r})\mathrm{d}p=\frac{\overline{\Delta}_v}{l}-H(\overline{r}-\overline{r}_b)\frac{\overline{w}(\overline{r})}{l} \quad (0\le\overline{r}\le 1) \tag{17}$$

其中 $l=\lim\limits_{p\to\infty}p\cdot f(p)=-(1-\nu)$, ν 为土骨架的泊松比，$H(p)=\dfrac{p\cdot f(p)}{l}-1$. 而圆板相对于刚核的挠度 $\overline{w}(\overline{r})$ 需满足经典薄板理论的微分方程

$$\left(\frac{\mathrm{d}}{\mathrm{d}\overline{r}^2}+\frac{1}{\overline{r}}\frac{\mathrm{d}}{\mathrm{d}\overline{r}}\right)^2\overline{w}(\overline{r})=-\delta\overline{\sigma}_{zw}(\overline{r},0) \quad (\overline{r}_b\le\overline{r}\le 1) \tag{18}$$

这里 $\delta=\dfrac{Gr_0^3}{D}$ (板的无量纲挠曲柔度).

通过变换

$$\overline{w}(\overline{r})=-\delta\int_0^\infty p\widetilde{\overline{\sigma}}_{zw}(p,0)\widetilde{w}(\overline{r},p)\mathrm{d}p \tag{19}$$

引入函数 $\widetilde{w}(\overline{r},p)$, 代入式 (18), 则 $\widetilde{w}(\overline{r},p)$ 应满足方程

$$\left(\frac{\mathrm{d}}{\mathrm{d}\bar{r}^2}+\frac{1}{\bar{r}}\frac{\mathrm{d}}{\mathrm{d}\bar{r}}\right)^2\widetilde{w}(\bar{r},p)=\mathrm{J}_0(p\bar{r}) \tag{20}$$

故

$$\widetilde{w}(\bar{r},p)=A_0+B_0\bar{r}^2+C_0\ln\bar{r}+D_0\bar{r}^2\ln\bar{r}+\frac{\mathrm{J}_0(p\bar{r})}{p^4} \tag{21}$$

式中四个任意常数 A_0,B_0,C_0,D_0 可由板变形的边界条件

$$[\overline{w}]_{\bar{r}=\bar{r}_b}=\left[\frac{\mathrm{d}\overline{w}}{\mathrm{d}\bar{r}}\right]_{\bar{r}=\bar{r}_b}=0,\quad \left[\frac{\mathrm{d}^3\overline{w}}{\mathrm{d}\bar{r}^3}+\frac{1}{\bar{r}}\frac{\mathrm{d}^2\overline{w}}{\mathrm{d}\bar{r}^2}\right]_{\bar{r}=1}=0,\quad \left[\frac{\mathrm{d}^2\overline{w}}{\mathrm{d}\bar{r}^2}+\frac{\nu_f}{\bar{r}}\frac{\mathrm{d}\overline{w}}{\mathrm{d}\bar{r}}\right]_{\bar{r}=1}=0 \tag{22}$$

得到. 故

$$\begin{aligned}\widetilde{w}(\bar{r},p)=&\frac{1}{p^4}[\mathrm{J}_0(p\bar{r})-\mathrm{J}_0(p\bar{r}_b)]+\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b}\frac{\mathrm{J}_1(p\bar{r}_b)}{p^3}+e_0\left(\ln\frac{\bar{r}}{\bar{r}_b}-\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b^2}\right)\times\\&\left\{(1+\nu_f)\left[\frac{\mathrm{J}_1(p\bar{r}_b)}{\bar{r}_bp^3}-\frac{\mathrm{J}_1(p)}{p^3}\right]+\frac{\mathrm{J}_2(p)}{p^2}-\frac{\mathrm{J}_1(p)}{2p}\right\}-\\&\left[\bar{r}^2\ln\frac{\bar{r}}{\bar{r}_b}-\frac{\bar{r}^2-\bar{r}_b^2}{2}-2e_0(1+\nu_f)\left(\ln\frac{\bar{r}}{\bar{r}_b}-\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b^2}\right)\ln\bar{r}_b\right]\frac{\mathrm{J}_1(p)}{4p}\end{aligned} \tag{23}$$

其中 $e_0=\dfrac{\bar{r}_b^2}{1+\bar{r}_b^2+\nu_f(1-\bar{r}_b^2)}$.

结合式 (17), (19), 再由 $\bar{\sigma}_{zw}(\bar{r},0)=\int_0^\infty p\tilde{\bar{\sigma}}_{zw}^0(p,0)\mathrm{J}_0(p\bar{r})\mathrm{d}p$ 我们可得一组描述混合边值问题的对偶积分方程

$$\int_0^\infty p^{-1}B(p)(1+H(p))\mathrm{J}_0(p\bar{r})\mathrm{d}p=\frac{\overline{\Delta}_v}{l}+\int_0^\infty\frac{\delta}{l}H(\bar{r}-\bar{r}_b)B(p)\widetilde{w}(\bar{r},p)\mathrm{d}p\quad(0\le\bar{r}\le1) \tag{24}$$

$$\int_0^\infty B(p)\mathrm{J}_0(p\bar{r})\mathrm{d}p=0\qquad(\bar{r}>1) \tag{25}$$

令

$$B(p)=\frac{2}{\pi}\frac{\overline{\Delta}_v}{l}p\int_0^1\theta(t)\cos pt\mathrm{d}t \tag{26}$$

代入式 (25) 则该式自动满足, 而式 (24) 可化为第二类 Fredholm 积分方程

$$\theta(x)+\int_0^1K(x,t)\theta(t)\mathrm{d}t=1 \tag{27}$$

式中核函数

$$K(x,t)=\int_0^\infty\frac{2}{\pi}H(p)\cos(px)\cos(pt)\mathrm{d}p-\frac{2\delta}{l\pi}\int_0^\infty p\cos(pt)\left\{\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x\frac{\bar{r}H(\bar{r}-\bar{r}_b)\widetilde{w}(\bar{r},p)}{\sqrt{x^2-\bar{r}^2}}\mathrm{d}\bar{r}\right\}\mathrm{d}p$$

它可进一步写成 (详见附录)

$$K(x,t)=\int_0^\infty \frac{2}{\pi}H(p)\cos(px)\cos(pt)\mathrm{d}p \quad (0<x<\overline{r}_b \text{ 或 } 0<t<\overline{r}_b) \tag{28a}$$

$$\begin{aligned}K(x,t)=&\int_0^\infty \frac{2}{\pi}H(p)\cos(px)\cos(pt)\mathrm{d}p-\\&\frac{\delta}{\pi l}\left\{(x^2+t^2)\ln\left(\frac{\overline{r}_b\sqrt{|x^2-t^2|}}{x\sqrt{t^2-\overline{r}_b^2}+t\sqrt{x^2-\overline{r}_b^2}}\right)+2xt\ln\left(\frac{\sqrt{x^2-\overline{r}_b^2}+\sqrt{t^2-\overline{r}_b^2}}{\sqrt{|x^2-t^2|}}\right)+\right.\\&\frac{xt}{\overline{r}_b^2}\sqrt{x^2-\overline{r}_b^2}\sqrt{t^2-\overline{r}_b^2}-e_0(1+\nu_f)\left[\ln\left(\frac{x+\sqrt{x^2-\overline{r}_b^2}}{\overline{r}_b^2}\right)-\frac{x\sqrt{x^2-\overline{r}_b^2}}{\overline{r}_b^2}\right]\times\\&\left.\left[\ln\left(\frac{t+\sqrt{t^2-\overline{r}_b^2}}{\overline{r}_b^2}\right)-\frac{t\sqrt{t^2-\overline{r}_b^2}}{\overline{r}_b^2}\right]\right\} \quad (\overline{r}_b\le x\le 1 \text{ 且 } \overline{r}_b\le t\le 1)\end{aligned} \tag{28b}$$

含刚核弹性圆板的平衡条件为

$$P=-\int_0^{r_0}\sigma_{zw}(r,0)\cdot 2\pi r\mathrm{d}r=-2\pi G r_0^2\widetilde{\overline{\sigma}}_{zw}^0(0,0) \tag{29}$$

将式 (26) 代入上式我们就得到

$$P=-\frac{4Gr_0\Delta_v}{l}\int_0^1\theta(t)\mathrm{d}t=\frac{4Gr_0}{1-\nu}[K_{vv}(a_0)+\mathrm{i}a_0C_{vv}(a_0)]\Delta_v \tag{30}$$

这里引入的 $K_{vv}(a_0),C_{vv}(a_0)$ 分别为饱和地基上含刚核弹性板竖向振动的动力刚度系数和阻尼系数，其中 $K_{vv}=\int_0^1\mathrm{Re}\,[\theta(t)]\mathrm{d}t$, $C_{vv}=\frac{1}{a_0}\int_0^1\mathrm{Im}\,[\theta(t)]\mathrm{d}t$.

饱和地基上刚性板与弹性板的竖向振动均为本文的特殊情形. 对于刚性板，只需在式 (28b) 中令 $\delta=0$, 则有 $K(x,t)=\int_0^\infty\frac{2}{\pi}H(p)\cos(px)\cos(pt)\mathrm{d}p$, 这和文献 [5] 结果一致. 而对于弹性板，在式 (23) 中令 $\overline{r}_b$ 趋于零，则有 $\lim\limits_{\overline{r}_b\to 0}\widetilde{w}(\overline{r},p)=\frac{1}{p^4}\mathrm{J}_0(p\overline{r})-\frac{1}{p^4}+\frac{\overline{r}^2}{4p^2}+\frac{\overline{r}^2}{2(1+\nu_f)}\left\{-\frac{1+\nu_f}{2p^2}+\frac{\mathrm{J}_1'(p)}{p^2}+\nu_f\frac{\mathrm{J}_1(p)}{p^3}\right\}+\frac{\overline{r}^2}{2(1+\nu_f)}\frac{\mathrm{J}_1(p)}{2p}+\frac{\overline{r}^2}{8p}\mathrm{J}_1(p)-\frac{1}{4p}\overline{r}^2\ln\overline{r}\mathrm{J}_1(p)$, 代入式 (24) 可发现描述混合边值问题的对偶积分方程和文献 [6] 完全一致，因此弹性板亦可视为本文的极限状态.

3 数值算例

解第二类 Fredholm 积分方程式 (27), 并结合式 (30), 我们可方便地得出饱和地基上含刚核弹性圆板竖向振动的动力刚度系数 K_{vv} 及阻尼系数 C_{vv} 随无量纲频率 a_0 的变化曲线. 在本节给出的数值算例中，饱和土体的物理力学参数为: $\overline{\lambda}=1$, $\nu=0.25$, $n=0.375$, $\overline{\rho}_1=0.1845$, $\overline{\rho}_2=0.8155$, $\overline{E}_w=11.489$. 我们对 $\overline{b}=2$, 300, 10^4 三种情况进行了验算，而每一个 $\overline{b}$ 值又分别考虑 $\overline{r}_b=0.25$, 0.5, 0.75; $\delta=1$, 10, 100, 1000 以考察板刚域大小及其挠曲柔度的影响，板的泊松比 $\nu_f=0.167$, 计算结果如图 2～ 图 4 所示.

由图 2～ 图 4 可见，当频率较低时，含刚核弹性板的动力刚度系数 K_{vv} 要低于刚性板的动力刚度系数，且其值随板的柔度的增加而逐渐减小；而在高频区域， $\overline{b}$ 值较大时 ($\overline{b}\ge 300$), 含刚核弹性板的动力刚度系数要比刚性板的大，当 $\overline{b}$ 值较小时 ($\overline{b}=2$), 刚性板的动力刚度系数介

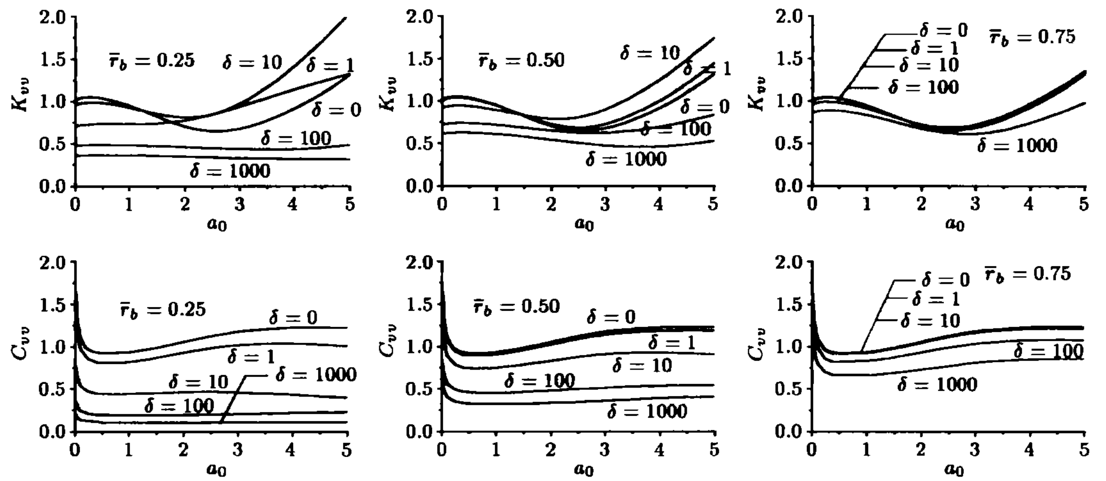

图 2 $\bar{b}=2$ 时含刚核弹性板振动的阻抗函数

Fig.2 Vertical impedance functions for the vibration of the elastic circular plate with rigid core ($\bar{b}=2$)

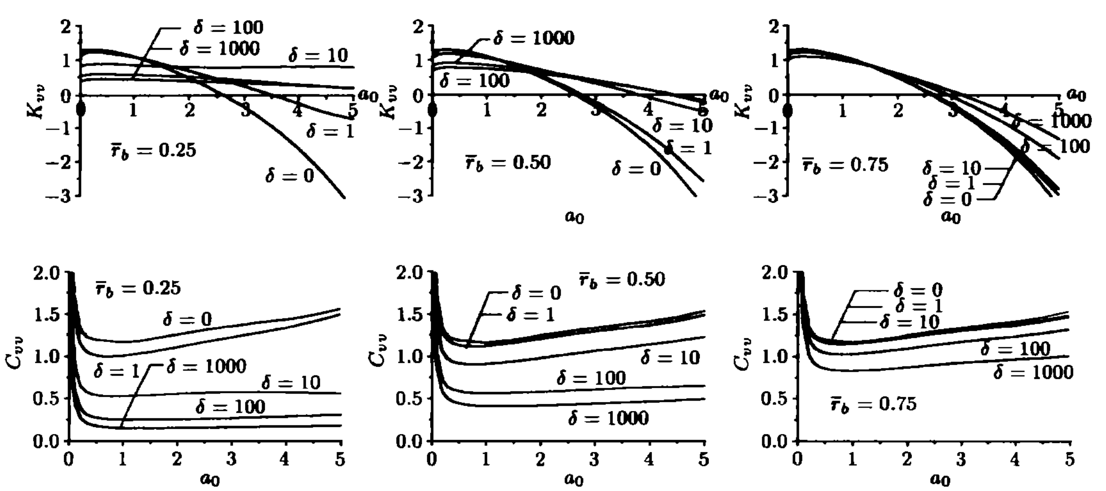

图 3 $\bar{b}=300$ 时含刚核弹性板振动的阻抗函数

Fig.3 Vertical impedance functions for the vibration of the elastic circular plate with rigid core ($\bar{b}=300$)

于 $\delta=10$ 和 $\delta=100$ 的含刚核弹性板之间. 此外, 在通常的 $\bar{b}$ 的取值范围内, 含刚核弹性板的动力阻尼系数 C_{vv} 明显地小于刚性板的值.

从图 2~ 图 4 还可发现, 随板的柔度的增加, 含刚核弹性板的动力刚度系数和阻尼系数值逐渐变得与频率无关, 而刚核尺寸的增加 ($\bar{r}_b$ 增加) 亦使板的柔度的影响逐渐减弱.

为验证本文理论推导的正确性, 我们计算了 $\bar{b}=2$, $\delta=10$, $\nu_f=0.167$, $\bar{r}_b=0.01$ 时饱和地基上含刚核弹性板振动的动力柔度系数, 其结果与参数相同的完全弹性板的动力柔度系数 (按

文献 [6] 的计算方法) 一并于表 1 给出. 由表 1 可见两组数据非常吻合, 因此当 $\bar{r}_b \to 0$ 时, 本文的结果可退化为弹性板振动的情形.

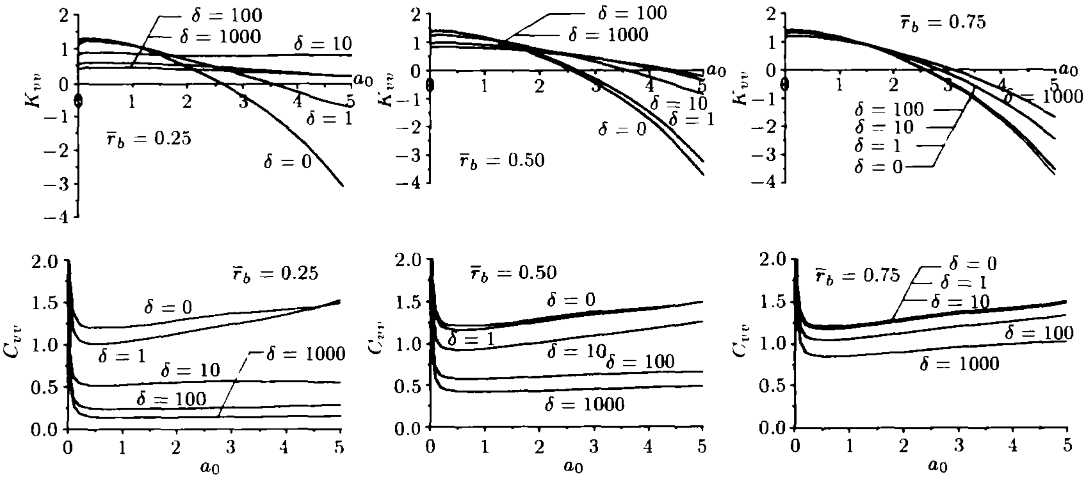

图 4 $\bar{b} = 10000$ 时含刚核弹性板振动的阻抗函数

Fig.4 Vertical impedance functions for the vibration of the elastic circular plate with rigid core ($\bar{b} = 10\,000$)

表 1 两种不同计算方法所得的动力柔度系数

Table 1 An comparison between the dynamic compliance coefficients obtained from two different methods

a_0	0.1	1	2	3	4	5
flexible plate with with a rigid core	$1.884 - 0.105j$	$1.531 - 0.6411j$	$1.051 - 0.7605j$	$0.8012 - 0.6838j$	$0.6689 - 0.6261j$	$0.568 - 0.5819j$
fully flexible plate	$1.892 - 0.105j$	$1.540 - 0.6415j$	$1.063 - 0.7619j$	$0.8147 - 0.6868j$	$0.6840 - 0.6275j$	$0.5885 - 0.5779j$

4 结　语

本文通过对饱和地基上含刚核弹性圆板的振动的研究, 初步可得以下几点结论:

(1) 当 $\bar{b} \geq 300$ 时, 其值对饱和地基上含刚核弹性圆板的振动的影响可忽略不计.

(2) 当 $\delta = 0$ 或 $\bar{r}_b \to 0$ 时, 本文的结果即为刚性板和弹性板的极限情形.

(3) 随板的挠曲柔度的增加, 含刚核弹性板的动力刚度系数和阻尼系数值逐渐变得与频率无关, 而刚核尺寸的增加 ($\bar{r}_b$ 增加) 亦使板的柔度的影响逐渐减弱.

附　录

这里不妨假设 $t \geq x \geq \bar{r}_b$, 记

$$\widetilde{w}(\bar{r}, p) = w_1(\bar{r}, p) + w_2(\bar{r}, p) + w_3(\bar{r}, p) + w_4(\bar{r}, p) + w_5(\bar{r}, p) \tag{A-1}$$

其中

$$w_1(\bar{r},p)=\frac{1}{p^4}[\mathrm{J}_0(p\bar{r})-\mathrm{J}_0(p\bar{r}_b)]+\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b}\frac{\mathrm{J}_1(p\cdot\bar{r}_b)}{p^3}$$

$$w_2(\bar{r},p)=e_0\left(\ln\frac{\bar{r}}{\bar{r}_b}-\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b^2}\right)(1+\nu_f)\left(\frac{\mathrm{J}_1(p\bar{r}_b)}{\bar{r}_b p^3}-\frac{\mathrm{J}_1(p)}{p^3}\right)$$

$$w_3(\bar{r},p)=e_0\left(\ln\frac{\bar{r}}{\bar{r}_b}-\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b^2}\right)\left(\frac{\mathrm{J}_2(p)}{p^2}-\frac{\mathrm{J}_1(p)}{2p}\right)$$

$$w_4(\bar{r},p)=2e_0(1+\nu_f)\ln\bar{r}_b\left(\ln\frac{\bar{r}}{\bar{r}_b}-\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b^2}\right)\frac{\mathrm{J}_1(p)}{4p}$$

$$w_5(\bar{r},p)=-\left(\bar{r}^2\ln\frac{\bar{r}}{\bar{r}_b}-\frac{\bar{r}^2-\bar{r}_b^2}{2}\right)\frac{\mathrm{J}_1(p)}{4p}$$

因

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{\bar{r}_b}^{x}\frac{\bar{r}}{\sqrt{x^2-\bar{r}^2}}\left[\frac{\mathrm{J}_0(\bar{r}p)-\mathrm{J}_0(\bar{r}_b p)}{p^3}+\frac{\bar{r}^2-\bar{r}_b^2}{2\bar{r}_b}\frac{\mathrm{J}_1(\bar{r}_b p)}{p^2}\right]\mathrm{d}\bar{r}=$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{\bar{r}_b}^{x}\sqrt{x^2-\bar{r}^2}\left\{\frac{-\mathrm{J}_1(p\bar{r})}{p^2}+\frac{\bar{r}\mathrm{J}_1(p\bar{r}_b)}{\bar{r}_b p^2}\right\}\mathrm{d}\bar{r}=\frac{x}{p}\int_{\bar{r}_b}^{x}\sqrt{x^2-\bar{r}^2}\frac{\mathrm{J}_2(\bar{r}p)}{\bar{r}}\mathrm{d}\bar{r} \tag{A-2}$$

$$\int_0^\infty\left[\frac{\mathrm{J}_1(\bar{r}_b p)}{\bar{r}_b p^2}-\frac{\mathrm{J}_1(p)}{p^2}\right]\cos(pt)\mathrm{d}p=\int_{\bar{r}_b}^{1}\frac{1}{\bar{r}_b}\left\{\int_0^\infty\frac{1}{p}\mathrm{J}_2(p\bar{r}_b)\cdot\cos(pt)\mathrm{d}p\right\}\mathrm{d}\bar{r}_b=$$

$$\begin{cases}-\dfrac{1}{2}\left\{\dfrac{t^2(1-\bar{r}_b^2)}{\bar{r}_b^2}+\ln\bar{r}_b\right\} & (t<\bar{r}_b)\\[2ex] -\dfrac{1}{2}\left\{\dfrac{t^2\left(1-\bar{r}_0^2\right)}{\bar{r}_b^2}+\ln\bar{r}_b-\dfrac{t\sqrt{t^2-\bar{r}_b^2}}{\bar{r}_b^2}-\ln\dfrac{\bar{r}_b}{t+\sqrt{t^2-\bar{r}_b^2}}\right\} & (t>\bar{r}_b)\end{cases} \tag{A-3}$$

故有

$$-\frac{2\delta}{\pi l}\int_0^\infty p\cos(pt)\left\{\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x\frac{\bar{r}H(\bar{r}-\bar{r}_b)w_1(\bar{r},p)}{\sqrt{x^2-\bar{r}^2}}\mathrm{d}\bar{r}\right\}\mathrm{d}p=$$

$$-\frac{2\delta}{\pi l}\int_0^\infty\frac{x}{p}\cos(pt)\int_{\bar{r}_b}^{x}\sqrt{x^2-\bar{r}^2}\frac{\mathrm{J}_2(\bar{r}p)}{\bar{r}}\mathrm{d}\bar{r}\mathrm{d}p=$$

$$-\frac{\delta}{\pi l}\left\{\left[t^2\ln\frac{x+\sqrt{x^2-\bar{r}_b^2}}{\bar{r}_b}-\frac{xt^2\sqrt{x^2-\bar{r}_b^2}}{\bar{r}_b^2}\right]+\left[x^2\ln\frac{x+\sqrt{x^2-\bar{r}_b^2}}{\bar{r}_b}-x\sqrt{x^2-\bar{r}_b^2}\right]+\right.$$

$$\left[\frac{xt}{\bar{r}_b^2}\sqrt{x^2-\bar{r}_b^2}\sqrt{t^2-\bar{r}_b^2}+(x^2+t^2)\ln\frac{\bar{r}_b\sqrt{t^2-x^2}}{x\sqrt{t^2-\bar{r}_b^2}+t\sqrt{x^2-\bar{r}_b^2}}+\right.$$

$$\left.\left.2xt\ln\frac{\sqrt{t^2-\bar{r}_b^2}+\sqrt{x^2-\bar{r}_b^2}}{\sqrt{t^2-x^2}}\right]\right\} \tag{A-4}$$

$$-\frac{2\delta}{\pi l}\int_0^\infty p\cos(pt)\left\{\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x\frac{\bar{r}H(\bar{r}-\bar{r}_b)w_2(\bar{r},p)}{\sqrt{x^2-\bar{r}^2}}\mathrm{d}\bar{r}\right\}\mathrm{d}p=-\frac{\delta}{\pi l}\cdot$$

$$2e_0(1+\nu_f)\left(-\frac{1}{2}\right)\left\{\frac{t^2\left(1-\bar{r}_b^2\right)}{\bar{r}_b^2}+\ln\bar{r}_b-\frac{t\sqrt{t^2-\bar{r}_b^2}}{\bar{r}_b^2}-\ln\frac{\bar{r}_b}{t+\sqrt{t^2-\bar{r}_b^2}}\right\}f(x) \tag{A-5}$$

这里

$$f(x)=\frac{\mathrm{d}}{\mathrm{d}x}\int_{\overline{r}_b}^{x}\frac{\overline{r}}{\sqrt{x^2-\overline{r}^2}}\left(\ln\frac{\overline{r}}{\overline{r}_b}-\frac{\overline{r}^2-\overline{r}_b^2}{2\overline{r}_b^2}\right)\mathrm{d}\overline{r}=\ln\frac{x+\sqrt{x^2-\overline{r}_b^2}}{\overline{r}_b}-\frac{x\sqrt{x^2-\overline{r}_b^2}}{\overline{r}_b}$$

$$-\frac{2\delta}{\pi l}\int_0^{\infty}p\cos(pt)\left\{\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{x}\frac{\overline{r}H(\overline{r}-\overline{r}_b)w_3(\overline{r},p)}{\sqrt{x^2-\overline{r}^2}}\mathrm{d}\overline{r}\right\}\mathrm{d}p=-\frac{\delta}{\pi l}(-2)e_0t^2f(x) \tag{A-6}$$

$$-\frac{2\delta}{\pi l}\int_0^{\infty}p\cos(pt)\left\{\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{x}\frac{\overline{r}H(\overline{r}-\overline{r}_b)w_4(\overline{r},p)}{\sqrt{x^2-\overline{r}^2}}\mathrm{d}\overline{r}\right\}\mathrm{d}p=-\frac{\delta}{\pi l}e_0(1+\nu_f)\ln\overline{r}_bf(x) \tag{A-7}$$

$$-\frac{2\delta}{\pi l}\int_0^{\infty}p\cos(pt)\left\{\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{x}\frac{\overline{r}H(\overline{r}-\overline{r}_b)w_5(\overline{r},p)}{\sqrt{x^2-\overline{r}^2}}\mathrm{d}\overline{r}\right\}\mathrm{d}p=-\frac{\delta}{\pi l}\left(x\sqrt{x^2-\overline{r}_b^2}+x^2\ln\frac{x-\sqrt{x^2-\overline{r}_b^2}}{\overline{r}_b}\right) \tag{A-8}$$

综合式 (A-4)~(A-8) 可得式 (28b) (见正文). 对于其它情形的 x,t 值, 我们亦可方便地得出.

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VERTICAL VIBRATIONS OF FLEXIBLE PLATE WITH RIGID CORE ON SATURATED GROUND[1)]

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Abstract A study of the vertical vibration of a flexible plate with rigid core resting on saturated ground is presented in this paper. The analysis is based on the theory of elastic wave in saturated soil established by the second author of the paper. We start with a derivation of the governing different equations for saturated soil by the use of Hankel transform and the corresponding Hankel transformed displacement solutions are obtained. Then, under the assumption of relaxed contact conditions, i.e., frictionless contact between the plate and the surface of the saturated ground and keeping in mind that the vertical displacement of the internal rigid core of plate be a constant while the outer flexible plate must satisfy the equation of motion of the classic thin plate, an dual integral equations which describe the mixed boundary-value problem for the flexible plate with a rigid core are established which are reduced to the second kind Fredholm integral equation and solved by standard procedure. At the end of this paper, the dynamic impedance functions for vertical vibration of the plate bearing on saturated ground are derived, the effects of the permeability, the size of the rigid core and the flexibility of the plate on the impedance functions are studied. The numerical results show that at the limit case of $\delta = 0$ ($\delta =$ the dimensionless flexural compliance of the plate) or $\overline{r}_b \to 0$ ($\overline{r}_b$ corresponds to the size of rigid core), the dynamic impedance functions are in exactly agreement with those for rigid plate and fully flexible plate respectively.

Key words saturated ground, circular flexible plate with a rigid core, vibration, dynamic impedence function, the second kind Fredholm integral equation, Hankel transform

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