第 33 卷 第 5 期 力 学 学 报 Vol. 33, No.5
2001 年 9 月 ACTA MECHANICA SINICA Sep., 2001

黏弹性圆柱形壳动力学高余维分岔、普适开折问题[1)]

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摘要 讨论两端受到谐波激励的黏弹性圆柱形壳的非线性动力学行为, 利用奇异性理论, 研究了分岔方程的普适开折问题, 严格证明了它是一个高余维分岔问题. 余维数为 5(含有一个模参数), 给出了它的所有可能的普适开折形式. 在分岔参数满足某些条件时得到该分岔问题的转迁集及分岔图, 展示了一些新的动力学行为, 改进和完善了奇异性分析方法.

关键词 黏弹性, 圆柱形壳, 分岔, 普适开折, 芽

引 言

众所周知, 国内外许多学者都曾对具有某种特殊形式的非线性 Mathieu 方程的动力学行为进行过深入研究, 如 Bogoliubov N.N. 和 Mitropolsky Y.A.[1], Nayfeh A.H. 和 Mook D.T.[2] 等, 特别在文 [3] 中, 陈予恕和 Langford W.F. 就一般形式的非线性 Mathieu 系统, 引入群对称技巧, 建立了该类系统的稳态周期响应分类结果, 成功地统一了文 [1, 2] 中的结论, 文 [3] 中给出的方法是确定稳态响应拓扑类型的重要方法. 此后, 陈予恕等开展了一系列单自由度参数激励系统的理论和实验研究, 讨论了此类系统的 Z_2 对称分岔问题 (见文献 [5~7]).

分岔方程的对称开折描述系统受到对称扰动时可能出现的动力学行为, 当系统受到非对称扰动时, 对称开折 (即使是普适的) 也不能全面刻画系统的动力学行为, 此时必须考虑一般意义下 (含对称破缺) 分岔方程的普适开折问题. 基于此, 本文利用奇异性理论, 严格证明了文 [7, 8] 中分岔方程的普适开折是一高余维分岔问题, 其余维数为 5, 得到了它的所有可能形式的普适开折, 考虑了普适开折的分类问题, 在开折参数适合某些条件时, 给出了转迁集及相应的分岔图. 所得结果比文 [7, 8] 中结果更全面, 且改进和完善了文 [3] 中提出的奇异性分析方法.

1 振幅分岔方程的普适开折

对黏弹性薄圆柱壳来说, 在满足 Kirchhoff-Love 假设下, 若考虑 Flugge 型几何非线性而忽略惯性耦合, 则其运动方程可写为 [9]

$$[N_{\alpha\beta}(\delta_{1\beta}+u_{1,\beta})]_{,\alpha}-Pu_{1,11}-\rho h u_{1,tt}=0$$

$$\left[N_{\alpha\beta}\left(\delta_{2\beta}\left(1+\frac{u_3}{R}\right)u_{2,\beta}\right)\right]_{,\alpha}-\frac{N_{2\beta}}{R}\left(u_{3,\beta}-\delta_{2\beta}\frac{u_2}{R}\right)+\frac{M_{\alpha 2,\alpha}}{R}-Pu_{2,11}-\rho h u_{2,tt}=0$$

2000–01–03 收到第一稿, 2001–03–19 收到第三稿.
1) 国家自然科学基金 (重大 19990510) 及国家重点基础研究基金 (G1998020316) 资助项目.

$$\left[N_{\alpha\beta}\left(u_{3,\beta}-\delta_{2\beta}\frac{u_2}{R}\right)\right]_{,\alpha}+\frac{N_{2\beta}}{R}\left[u_{2,\beta}+\delta_{2\beta}\left(1+\frac{u_3}{R}\right)\right]+M_{\alpha\beta,\alpha\beta}-Pu_{3,11}-\rho h u_{3,tt}=0$$

文 [8] 指出：在两端简支或夹支条件下，该系统的 Galerkin 离散方程的中心流形上的方程可用单自由度参数激励系统

$$\frac{\mathrm{d}^2u}{\mathrm{d}t^2}+\overline{\delta}q(u)\frac{\mathrm{d}u}{\mathrm{d}t}+(m^2+\lambda)u+\alpha u^2+\beta u^3-2\varepsilon\cos(kt)u=0$$

来描述，并用 LS 方法导出了稳态非平凡周期解振幅分岔方程

$$Dr^4+6\lambda\beta r^2+2m^2\overline{\delta}^2q_0q_2r^2-\varepsilon^2\frac{\xi}{6m^2}\left(\frac{\alpha^2}{m^2}+\frac{3\beta}{2}\right)r^2+\lambda^2+m^2\overline{\delta}^2q_0^2-\varepsilon^2=0 \tag{1}$$

这里 $\xi=\cos 4\theta$, $D=9\beta^2+m^2\overline{\delta}^2q_2^2-\dfrac{\varepsilon^2}{144m^4}\left(\dfrac{\alpha^2}{m^2}+\dfrac{3\beta}{2}\right)$, 当 $D\neq 0$ 时，引入非线性变换

$$\phi x^4=|D|r^4,\quad \phi=\mathrm{sgn}\,(D)$$

则分岔方程 (1) 成为

$$\phi x^4+2\eta x^2\lambda+\lambda^2+\alpha_1x^2+\alpha_2=0 \tag{2}$$

其中 $\eta=\dfrac{3\beta}{\sqrt{|D|}}$, $\alpha_1=\dfrac{1}{\sqrt{|D|}}\left[2m^2\overline{\delta}^2q_0q_2-\dfrac{\varepsilon^2\xi}{6m^2}\left(\dfrac{\alpha^2}{m^2}+\dfrac{3\beta}{2}\right)\right]$, $\alpha_2=m^2\overline{\delta}^2q_0^2-\varepsilon^2$.

文 [8] 视 λ 为分岔参数， α_1,α_2 为开折参数，讨论了分岔方程 (2) 的开折问题，所得结果是不完备的，本文使用奇异性理论，研究分岔方程 (2) 的普适开折问题.

取芽 $g(x,\lambda)=\phi x^4+2\eta x^2\lambda+\lambda^2$, 我们讨论 $\phi=1$, $|\eta|\neq 1$ 的非退化情形 (退化情形 $|\eta|=1$, 芽 g 有无穷余维，详细参见文献 [3, 10]). 有下列结果:

定理 1 当 $\eta\neq 0$ 时，芽 $g(x,\lambda)$ 的限制切空间为

$$RT(g)=\mathcal{M}^5+\mathcal{M}^3\langle\lambda\rangle+\mathcal{M}\langle\lambda^2\rangle+R\{g,xg_x\} \tag{3}$$

证明 分四步完成.

(1) 该部分证明

$$\mathcal{M}^5\subset RT(g) \tag{4}$$

由限制切空间 $RT(g)$ 的表达式得

$$RT(g)=\langle g,xg_x,\lambda g_x\rangle=$$
$$\langle x^4+2\eta x^2\lambda+\lambda^2,4x^4+4\eta x^2\lambda,4x^3\lambda+4\eta x\lambda^2\rangle$$

要证明 $\mathcal{M}^5\subset RT(g)$, 据 Nakayama 引理，只需证明 $\mathcal{M}^5\subset RT(g)+\mathcal{M}^6$ 即可.

事实上，考虑到 $\mathcal{M}^5$ 的生成子为： $x^5,x^4\lambda,x^3\lambda^2,x^2\lambda^3,x\lambda^4,\lambda^5$, 且

$$x^5=\frac{\eta}{4(1-\eta^2)}\left[4\eta x\cdot g+\left(\frac{1}{\eta}-2\eta\right)x\cdot(xg_x)-\lambda g_x\right]\in RT(g)+\mathcal{M}^6$$

$$x^4\lambda=\frac{1}{4\eta}x^2\cdot(xg_x)-\frac{1}{\eta}x^6\in RT(g)+\mathcal{M}^6$$

$$x^3\lambda^2 = \frac{1}{4\eta}x\lambda\cdot(xg_x) - \frac{1}{\eta}x^5\lambda \in RT(g) + \mathcal{M}^6$$

$$x^2\lambda^3 = \frac{1}{4\eta}\lambda^2(xg_x) - \frac{1}{\eta}x^4\lambda^2 \in RT(g) + \mathcal{M}^6$$

$$x\lambda^4 = \frac{1}{4\eta}\lambda^2(\lambda g_x) - \frac{1}{\eta}x^3\lambda^3 \in RT(g) + \mathcal{M}^6$$

$$\lambda^5 = \lambda^3\cdot g - \lambda^3 x^4 - 2\eta x^2\lambda^4 \in RT(g) + \mathcal{M}^6$$

因此

$$\mathcal{M}^5 \subset RT(g) + \mathcal{M}^6$$

从而有式 (4) 成立.

(2) 该部分我们证明

$$\mathcal{M}^3\langle\lambda\rangle \subset RT(g) \tag{5}$$

$\mathcal{M}^3\langle\lambda\rangle$ 的生成子为： $x^3\lambda, x^2\lambda^2, x\lambda^3, \lambda^4$, 显然有 (注意到 (1) 中已证结果 $\mathcal{M}^5 \subset RT(g)$)

$$x^3\lambda = \frac{1}{4\eta}x\cdot(xg_x) - \frac{1}{\eta}x^5 \in RT(g) + \mathcal{M}^5 \subset RT(g)$$

$$x^2\lambda^2 = \frac{1}{4\eta}\lambda\cdot(xg_x) - \frac{1}{\eta}x^4\lambda \in RT(g) + \mathcal{M}^5 \subset RT(g)$$

$$x\lambda^3 = \frac{1}{4\eta}\lambda\cdot(\lambda g_x) - \frac{1}{\eta}x^3\lambda^2 \in RT(g) + \mathcal{M}^5 \subset RT(g)$$

$$\lambda^4 = \lambda^2\cdot g - x^4\lambda^2 - 2\eta x^2\lambda^3 \in RT(g) + \mathcal{M}^5 \subset RT(g)$$

即式 (5) 成立.

(3) 该部分证明

$$\mathcal{M}\langle\lambda^2\rangle \subset RT(g) \tag{6}$$

$\mathcal{M}\langle\lambda^2\rangle$ 的生成子为： $x\lambda^2, \lambda^3$, 我们有

$$x\lambda^2 = \frac{1}{4\eta}\cdot\lambda g_x - \frac{1}{\eta}x^3\lambda \in RT(g) + \mathcal{M}^3\langle\lambda\rangle \subset RT(g)$$

$$\lambda^3 = \lambda\cdot g - x^4\lambda - 2\eta x^2\lambda^2 \in RT(g) + \mathcal{M}^3\langle\lambda\rangle \subset RT(g)$$

于是式 (6) 成立.

考虑到 $R\{g, xg_x\} \subset RT(g)$, 结合式 (4)~(6) 得

$$\mathcal{M}^5 + \mathcal{M}^3\langle\lambda\rangle + \mathcal{M}\langle\lambda^2\rangle + R\{g, xg_x\} \subset RT(g) \tag{7}$$

(4) 由限制切空间 $RT(g)$ 的定义得

$$RT(g) = \{ag + b(xg_x) + c(\lambda g_x) : a, b, c \in \varepsilon_{x,\lambda}\}$$

记 $a = a(0,0) + \overline{a}$, $b = b(0,0) + \overline{b}$; $\overline{a}, \overline{b} \in \mathcal{M}$

$$ag + b(xg_x) + c(\lambda g_x) = a(0,0)g + \overline{a}g + b(0,0)(xg_x) + \overline{b}(xg_x) + c(\lambda g_x)$$

不难知道, $xg, \lambda g, x\cdot(xg_x), \lambda\cdot(xg_x), \lambda g_x \in \mathcal{M}^5+\mathcal{M}^3\langle\lambda\rangle+\mathcal{M}\langle\lambda^2\rangle$, 于是得到

$$\bar{a}g+\bar{b}(xg_x)+c(\lambda g_x)\in \mathcal{M}^5+\mathcal{M}^3\langle\lambda\rangle+\mathcal{M}\langle\lambda^2\rangle$$

从而有

$$ag+b(xg_x)+c(\lambda g_x)\in \mathcal{M}^5+\mathcal{M}^3\langle\lambda\rangle+\mathcal{M}\langle\lambda^2\rangle+R\{g, xg_x\} \tag{8}$$

结合式 (7), (8) 便知式 (3) 成立. 证毕.

定理 2 $\eta\neq 0$ 时, 芽 g 的余维数为 5, 其普适开折分别为

$$G_1(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=(1+\alpha_0)x^4+2\eta x^2\lambda+\lambda^2+\alpha_1 x^3+\alpha_2 x^2+\alpha_3 x+\alpha_4 \tag{9}$$

$$G_2(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=(1+\alpha_0)x^4+2\eta x^2\lambda+\lambda^2+\alpha_1 x\lambda+\alpha_2 x^2+\alpha_3 x+\alpha_4 \tag{10}$$

$$G_3(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=x^4+(2\eta+\alpha_0)x^2\lambda+\lambda^2+\alpha_1 x^3+\alpha_2 x^2+\alpha_3 x+\alpha_4 \tag{11}$$

$$G_4(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=x^4+(2\eta+\alpha_0)x^2\lambda+\lambda^2+\alpha_1 x^3+\alpha_2 \lambda+\alpha_3 x+\alpha_4 \tag{12}$$

$$G_5(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=x^4+(2\eta+\alpha_0)x^2\lambda+\lambda^2+\alpha_1 x\lambda+\alpha_2 x^2+\alpha_3 x+\alpha_4 \tag{13}$$

$$G_6(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=x^4+(2\eta+\alpha_0)x^2\lambda+\lambda^2+\alpha_1 x\lambda+\alpha_2 \lambda+\alpha_3 x+\alpha_4 \tag{14}$$

证明 由切空间的表达式得

$$T(g)=RT(g)\oplus R\{g_x, g_\lambda, \lambda g_\lambda, \cdots, \lambda^l g_\lambda\} \tag{15}$$

由于 $\lambda g_\lambda\in RT(g)$, 可得

$$\begin{aligned} T(g)=RT(g)\oplus R\{g_x, g_\lambda\}= \\ \mathcal{M}^5+\mathcal{M}^3\langle\lambda\rangle+\mathcal{M}\langle\lambda^2\rangle\oplus R\{g, xg_x, g_x, g_\lambda\}= \\ \mathcal{M}^5+\mathcal{M}^3\langle\lambda\rangle+\mathcal{M}\langle\lambda^2\rangle\oplus R\{\eta x^2\lambda+\lambda^2, x^4+\eta x^2\lambda, x^3+\eta x\lambda, \eta x^2+\lambda\}\end{aligned}$$

不难知道

$$\mathrm{Itr}\, T(g)=\mathcal{M}^5+\mathcal{M}^3\langle\lambda\rangle+\mathcal{M}\langle\lambda^2\rangle$$

从而

$$[\mathrm{Itr}\, T(g)]^\perp=R\{x^4, x^3, x^2\lambda, x^2, x\lambda, \lambda^2, x, \lambda, 1\}$$

显然, 它的维数是 9, 由于 $R\{\eta x^2\lambda+\lambda^2, x^4+\eta x^2\lambda, x^3+\eta x\lambda, \eta x^2+\lambda\}$ 含有 4 个线性无关元素, 故它在 $[\mathrm{Itr}\, T(g)]^\perp$ 中的补基含 5 个线性无关元素, 故有 $\mathrm{codim}\,(g)=5$, 且不难知道各组补基分别为

x^4, x^3, x^2, x, 1;或 x^4, $x\lambda$, x^2, x, 1; 或 x^3, $x^2\lambda$, x^2, x, 1;
或 x^3, $x^2\lambda$, λ, x, 1; 或 $x^2\lambda$, $x\lambda$, x^2, x, 1; 或 $x^2\lambda$, $x\lambda$, λ, x, 1

显然各组补基对应的普适开折即是式 (9)~(14). 证毕.

定理 3 $\eta=0$ 时, 芽 g 的余维数也是 5, 此时不含模参数, 且普适开折是唯一的. 即

$$G(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=x^4+\lambda^2+\alpha_0x^2\lambda+\alpha_1x\lambda+\alpha_2x^2+\alpha_3x+\alpha_4 \tag{16}$$

证明 显然，芽 g 的限制切空间为

$$RT(g)=\langle g,xg_x,\lambda g_x\rangle=\langle x^4+\lambda^2,4x^4,4x^3\lambda\rangle=\langle\lambda^2,x^4,x^3\lambda\rangle$$

不难证明 $\mathcal{M}^4\subset RT(g)$ 且

$$RT(g)=\mathcal{M}^4\oplus R\{x\lambda^2,\lambda^3,\lambda^2\}$$

据 $T(g)=RT(g)\oplus R\{g_x,g_\lambda,\lambda g_\lambda,\cdots,\lambda^l g_\lambda\}$ 且 $\lambda g_\lambda=2\lambda^2\in RT(g)$，于是

$$T(g)=\mathcal{M}^4\oplus R\{x\lambda^2,\lambda^3,\lambda^2\}\oplus R\{x^3,\lambda\}=\mathcal{M}^4\oplus R\{x^3,x\lambda^2,\lambda^3,\lambda^2,\lambda\}$$

显然

$$[\mathrm{Itr}\,T(g)]^\perp=R\{x^3,x^2\lambda,x\lambda^2,\lambda^3,x^2,x\lambda,\lambda^2,x,\lambda,1\}$$

于是 $\mathrm{codim}\,(g)=5$，且 $R\{x^3,x\lambda^2,\lambda^3,\lambda^2,\lambda\}$ 在 $[\mathrm{Itr}T(g)]^\perp$ 中有唯一一组补基：$x^2\lambda,x^2,x\lambda,x,1$，此时 g 有唯一的普适开折，即

$$G(x,\lambda,\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=x^4+\lambda^2+\alpha_0x^2\lambda+\alpha_1x\lambda+\alpha_2x^2+\alpha_3x+\alpha_4$$

此即式 (16)．证毕．

当 $D=0$ 时，分岔方程 (1) 式为

$$6\beta x^2\lambda+\lambda^2+\alpha_1x^2+\alpha_2=0$$

我们考虑芽 $g(x,\lambda)\equiv 6\beta x^2\lambda+\lambda^2$．有下列结果．

定理 4 $D=0,\beta\neq 0$ 时，芽 g 有无穷余维．

证明 此时有

$$RT(g)=\langle g,xg_x,\lambda g_x\rangle=\langle\lambda^2,x^2\lambda,x\lambda^2\rangle=\langle x^2\lambda,\lambda^2\rangle$$
$$T(g)=\langle x^2\lambda,\lambda^2\rangle\oplus R\{x\lambda,6\beta x^2+2\lambda\}$$

显然 $\mathrm{codim}\,(g)=\infty$，且 $1,x,x^2,x^3,x^4,\cdots,x^n,\cdots$ 是一组补基．证毕．

2 普适开折的分类、转迁集与分岔图

我们注意到 $\eta\neq 0$ 时 $g(x,\lambda)$ 的普适开折 (13) 和 $\eta=0$ 时的普适开折 (16) 有相同的表达式，只不过 $\eta\neq 0$ 时，式 (13) 中开折参数 α_0 为模参数．本节我们取这个共同的普适开折形式来讨论分岔方程 (2) 的分岔行为．

$$G(x,\lambda,m,\alpha_1,\alpha_2,\alpha_3,\alpha_4)=x^4+mx^2\lambda+\lambda^2+\alpha_1x\lambda+\alpha_2x^2+\alpha_3x+\alpha_4 \tag{17}$$

2.1 芽的分类

视 m 为模参数将 $g(x,\lambda)=x^4+mx^2\lambda+\lambda^2=0$ 的解曲线进行分类：

I. 当 $|m|>2$ 时，对应两条相切于原点的抛物线 $\lambda=\dfrac{-m\pm\sqrt{m^2-4}}{2}x^2$；

II. 当 $|m|=2$ 时，对应一条顶点为原点的抛物线 $\lambda=-\frac{m}{2}x^2$;

III. 当 $|m|<2$ 时，无解.

对 $g(x,\lambda)$ 的开折，已有学者针对不同的情况进行过讨论．如文 [8] 讨论了双参数开折

$$x^4+mx^2\lambda+\lambda^2+\alpha_2x^2+\alpha_4$$

的情形 I 和 III; 情形 II 属无穷余维，详细参见文献 [3, 10].

普适开折 (17) 的分岔的详尽讨论比较困难．我们分析了 (17) 中包含的其它五种形式的双参数开折，得到了相应的转迁集和分岔图．限于篇幅，这里只给出有关

$$G(x,\lambda,\alpha_1,\alpha_4)=x^4+mx^2\lambda+\lambda^2+\alpha_1x\lambda+\alpha_4=0 \tag{18}$$

的结果.

2.2 转迁集和分岔图

由转迁集的定义，可求得开折 (18) 的转迁集为:

(a) 分岔集

$$B_1:\quad \alpha_4=0$$

$$B_2^{\pm}:\quad \alpha_4=\frac{1}{128}\frac{\left(m^4-80m^2-128\pm\sqrt{m^2(m^2+32)^3}\right)\alpha_1^4}{(m-2)^3(m+2)^3}$$

(b) 滞后集

$$H:\quad 256m^6\alpha_4+243(m^2+12){\alpha_1}^4=0$$

(c) 双极限点集

$$DL:\quad \alpha_1=0 \text{ 且} \begin{cases}\alpha_4>0, & \text{情形 I}\\ \alpha_4<0, & \text{情形 III}\end{cases}$$

考虑到 x 的物理意义，应有 $x\geq 0$, 此时开折 (18) 的转迁集变为:

情形 I:

(a1) 分岔集

$$B_1;\ B_2^{\pm} \text{ 且} \begin{cases}\alpha_1<0, & \text{当 } m>2\\ \alpha_1>0, & \text{当 } m<-2\end{cases}$$

(b1) 滞后集

$$H \text{ 且} \begin{cases}\alpha_1<0, & \text{当 } m>2\\ \alpha_1>0, & \text{当 } m<-2\end{cases};\quad \alpha_1=0 \text{ 且 } \alpha_4\leq 0$$

(c1) 双极限点集：空集.

情形 III:

(a3) 分岔集

$$B_1;\ B_2^{+} \text{ 且} \begin{cases}\alpha_1<0, & \text{当 } 0<m<2\\ \alpha_1>0, & \text{当 } -2<m<0\end{cases};\quad B_2^{-} \text{ 且} \begin{cases}\alpha_1>0, & \text{当 } 0<m<2\\ \alpha_1<0, & \text{当 } -2<m<0\end{cases}$$

(b3) 滞后集

$$H \text{ 且} \begin{cases} \alpha_1 < 0, & \text{当 } 0 < m < 2 \\ \alpha_1 > 0, & \text{当 } -2 < m < 0 \end{cases}$$

(c3) 双极限点集：空集.

由此可以看出，对状态变量所加的限制，不仅导致转迁集范围的变化，而且还导致转迁集类型的变化，如双极限点集 DL, 在限制条件 $x \geq 0$ 下，变成滞后集 H. 转迁集的这种变化，在用奇异性理论分析力学问题时，应给予特别的注意.

根据转迁集的上述表示，我们得到情形 I, III 的转迁集及相应的摄动保持分岔图 (见图 1 和图 2). 易知，对取定的 $|m| \neq 2$, 对应于 $\pm m$ 的转迁集关于 $\alpha_1 = 0$ 对称，标号相同区域中的摄动保持分岔图关于 $\lambda = 0$ 对称.

与对称开折的结果 (见文献 [8]) 比较可知，在形如 (18) 的非对称开折中，出现了新的动态分岔模式 I.3, I.4, I.6. 事实上， I.3 中两个解枝上都有极限点， I.4 的低解枝上有两个极限点， I.6 的低解枝上有一个极限点.

3 结 论

对轴向周期激励下具有 Flugge 型非线性黏弹性薄壳，本文严格证明了它的动态屈曲分岔是高余维分岔问题，其余维数为 5, 所有可能的普适开折形式共有 6 种.

选择与参数激励系统的分岔方程直接联系的普适开折 (13), 研究了它的对称破缺双参数开折的转迁集和分岔图，结果表明：非对称开折情形下，轴向周期激励黏弹性薄壳中存在比对称开折更为丰富的动态屈曲形式，这些新得到的动态屈曲形式为此类系统动态屈曲实验的分析和设计提供了启示.

本文中非退化芽的选取改进和完善了文献 [3] 中提出的奇异性分析方法，对于其它参数激励系统的动力学研究也有参考价值.

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THE HIGH CODIMENSIONAL BIFURCATIONS AND UNIVERSAL UNFOLDING PROBLEMS OF A VISCOELASTIC CIRCULAR CYLINDRICAL SHELL[1)]

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Abstract The nonlinear dynamical behavior of a viscoelastic circular cylindrical shell under a harmonic excitation applied at both ends is discussed. Using singularity theory and choosing nondegenerate germ, the universal unfolding problem of the bifurcation equation is studied. The results indicate that the universal unfolding is a high codimensional bifurcation problem with codimension 5 (contains a modal parameter). Additionally, all possible forms of the universal unfoldings are given, and the transition sets in parameter plane and the bifurcation diagrams are plotted under some conditions for unfolding parameters.

It is well know that the dynamical behavior of nonlinear Mathieu equation in some special forms has been studied by many authors, such as, Bogoliubov N.N., Mitropolsky Y.A., Nayfeh A.H. and Mook D.T. Especially, by introduction of symmetry group action technique, Chen Yushu and Langford W.F. established the bifurcation classification results for stable responses of nonlinear Mathieu equation in general form. A serise of theoretical and experimental results with Z_2-symmetric bifurcations were obtained for a parametric excitation system with one degree of freedom. One knows that a symmetric unfolding of a bifurcation equation may describe possible dynamical behavior when original system is subjected to a small perturbation with the symmetry. However, when original system is subjected to a small perturbation which does not possess the symmetry, symmetric unfolding cann't present completely the dynamical behavior of original system. In this case, one should investigate the universal unfoldings of bifurcation equation including symmetry-breaking bifurcations.

For a thin viscoelastic circular cylindrical shell with Flugge-type nonlinearity under a periodic excitation applied at axis. The present paper studies the symmetry-breaking bifurcation problem. The results obtained here indicate that, under symmetry-breaking cases, there exist richer dynamic buckling patterns than those obtained under symmetry case. Clearly, the new results provide some inspirations for the analysis and design of dynamic bulking experiments of this class of system, and improve the singularity analysis method proposed by Chen Yushu and Langford W.F.

Key words viscoelasticity, circular cylindrical shell, bifurcation, universal unfolding, germ

Received 3 January 2000, revised 19 March 2001.

1) The project supported by the National Natural Science Foundation of China (Significance 19990510) and the National Key Basic Research Special Foundation of China (G 1998020316).