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一种全耦合多相流分析的并行计算方法[1)]

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摘要 研究了孔隙介质中热、水和汽流全耦合分析的并行计算方法.模型中采用了考虑毛细压力关系的修正有效应力概念,并考虑了相变和潜热传递.基本变量为位移、毛细压力、汽压和温度.并行程序是在国家高性能计算中心(北京)的曙光 1000A 上借助 PVM(Parallel Virtual Machine)软件系统实现的,考题显示出较高的并行加速比和效率.

关键词 并行计算,多相流,可变形孔隙介质,相变

引 言

多相流问题的研究对于石油和天然气的开采,地热的开发,环境保护,海底隧道及大型地下建筑等工程领域具有十分重大的意义,成为近年来一个热门研究领域[1].由于问题的非线性以及求解的时间跨度较长,多相流分析具有高度计算复杂性,高速并行计算是解决这一困难的关键.然而,在多相流分析领域中有效并行算法的研究尚不多见.本文在分析该问题的内在并行性的基础上,提出一种适于非线性分析的并行波前算法,用于求解孔隙介质中包括热和质量传递的全耦合多相流问题.本算法是将问题的求解区域划分成一些子区域,在子区域内利用波前法[2,3]并行进行组装和内部自由度的消元过程.当每一个波前前进到各自的边界时,从波前活动变量核心数组提取相应系数组装界面方程.求解界面方程得出界面节点自由度值,再返回各子区域利用波前回代过程求出域内节点的自由度值.与一般基于带宽格式的区域分裂法相比,本方法不限制有限元节点的编号顺序,内存需要量少,编程容易.本算法已在国家高性能计算中心(北京)的曙光 1000A 上借助 PVM 系统予以实现,显示出较高的并行加速比和计算效率.

1 问题的数学模型

1.1 本构关系

将饱和度表示为毛细压力和温度的函数

$$S_w = S_w(p_c, T) \tag{1}$$

毛细压力 p_c 为

$$p_c = p_g - p_w \tag{2}$$

p_g, p_w 分别为潮湿空气和流体水压力,孔隙系统中的潮湿空气假定为干空气(ga)与水蒸汽(gw)两种理想气体的完全混合,服从理想气体状态方程.

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1)国家自然科学基金和国家高性能计算基金资助项目.

$$p_{ga} = \rho_{ga} TR/M_{ga}, \quad p_{gw} = \rho_{gw} TR/M_{gw}, \quad p_g = p_{ga} + p_{gw} \tag{3}$$

其中 M_π, ρ_π 分别是组分 π (干空气和水蒸汽) 的莫尔质量和密度, R 是气体常数, T 为绝对温度. 由 Kelvin-Laplace 定律, 孔隙中湿空气的相对湿度 (H) 为

$$H = \frac{p_{gw}}{p_{gws}} = \exp\left(-\frac{p_c M_w}{\rho_w RT}\right) \tag{4}$$

由 Clasius-Clapeyron 方程得水蒸汽饱和压力

$$p_{gws}(T) = p_{gws0}\exp\left[-\frac{M_w \Delta h_{vap}}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \tag{5}$$

其中 T_0 是初始温度, p_{gws0} 是 T_0 时的饱和汽压, Δh_{vap} 为气化热. 由修正的有效应力概念可导出固体相的本构关系

$$\sigma = \sigma' - \alpha(S_w p_w + S_g p_g) \mathrm{I} \tag{6}$$

σ 是总应力, σ' 是有效应力或 Bishop 应力, I 是单位张量, α 是 Biot 常数

$$\alpha = 1 - \frac{K_T}{K_S} \leq 1 \tag{7}$$

K_T 和 K_S 分别是孔隙介质和固体的体积模量. 有效应力 σ' 由固体骨架变形产生, 与应变的关系为

$$\mathrm{d}\sigma' = C_T(\mathrm{d}\varepsilon - \mathrm{d}\varepsilon^T - \mathrm{d}\varepsilon^0) \tag{8}$$

其中切向本构张量

$$C_T = C_T(\varepsilon, \sigma', T) \tag{9}$$

$\mathrm{d}\varepsilon^T = \mathrm{I}\frac{\beta_s}{3}\mathrm{d}T$ 是由热弹性膨胀引起的应变增量, β_s 是固体的体热胀系数. 由 Darcy 方程

$$v_\pi = -\frac{K K_{r\pi}}{\mu_\pi}(\mathrm{grad}\ p_\pi - \rho_\pi g) \tag{10}$$

其中 v 是 π (液和气) 相相对于固体相的速度, K 是绝对渗透系数, $K_{r\pi}$, μ_π 分别是 π 相的相对渗透系数和动粘度, g 是重力加速度. 扩散质量由 Fick 定律控制

$$v_{ga} = -\frac{M_a M_w}{M^2} D_{eff}\,\mathrm{grad}\left(\frac{p_{ga}}{p_g}\right) = \frac{M_a M_w}{M^2} D_{eff}\,\mathrm{grad}\left(\frac{p_{gw}}{p_g}\right) = -v_{gw} \tag{11}$$

Deff 是气体扩散系数.

1.2 控制方程

控制方程由运用均匀化方法和上述本构关系于微相方程得出[7,8]:

(1) 气体质量守恒方程

$$\frac{\partial}{\partial t}[\Phi(1 - S_w)\rho_{ga}] + a(1 - S_w)\rho_{ga}\mathrm{div}\,\dot{u} - \mathrm{div}\left[\rho_{ga}\frac{K K_{rg}}{\mu_g}\mathrm{grad}\ p_g\right] +$$

$$\mathrm{div}\left[\rho_g \frac{M_a M_w}{M^2} D_{\mathrm{eff}} \,\mathrm{grad}\left(\frac{p_{gw}}{p_g}\right)\right] = 0 \tag{12}$$

其中 $\dot{u}$ 是固体速度，ϕ 为孔隙率.

(2) 液体质量守恒方程

$$\frac{\partial}{\partial t}[\Phi(1 - S_w)\rho_{gw}] + \alpha(1 - S_w)\rho_{gw}\mathrm{div}\,\dot{u} - \mathrm{div}\left[\rho_{gw}\frac{K K_{rg}}{\mu_g}\,\mathrm{grad}\,p_g\right] -$$

$$\mathrm{div}\left[\rho_g \frac{M_a M_w}{M^2} D_{\mathrm{eff}}\,\mathrm{grad}\left(\frac{p_{gw}}{p_g}\right)\right] =$$

$$-\frac{\partial}{\partial t}(\Phi S_w \rho_w) - \alpha S_w \rho_w \mathrm{div}\,\dot{u} + \mathrm{div}\left[\rho_w \frac{K K_{rw}}{\mu_w}(\mathrm{grad}\,p_g - \mathrm{grad}\,p_c - \rho_w g)\right] \tag{13}$$

(3) 能量守恒方程

$$\rho C_p \frac{\partial T}{\partial t} - \mathrm{div}(\lambda_{\mathrm{eff}}\,\mathrm{grad}\,T) -$$

$$\left[C_{pw}\rho_w \frac{K K_{rw}}{\mu_w}(\mathrm{grad}\,p_g - \mathrm{grad}\,p_c - \rho_w g) + C_{pg}\rho_{gw}\frac{K K_{rg}}{\mu_g}\mathrm{grad}\,p_g\right]\cdot\mathrm{grad}\,T =$$

$$\Delta h_{\mathrm{vap}}\left[\frac{\partial}{\partial t}(\Phi S_w \rho_w) + \alpha S_w \rho_w \mathrm{div}\,\dot{u} - \mathrm{div}\left(\rho_w \frac{K K_{rw}}{\mu_w}(\mathrm{grad}\,p_g - \mathrm{grad}\,p_c - \rho_w g)\right)\right] \tag{14}$$

式中 Δh_{vap} 是汽化热.

(4) 线性矩平衡方程

$$\mathrm{div}[\sigma' - \mathrm{I}(p_g - S_w p_c)] + \rho g = 0 \tag{15}$$

(5) 初值条件

$$\text{当 } t = 0 \text{ 时 } p_g = p_g^0,\quad p_c = p_c^0,\quad T = T^0,\quad u = u^0 \tag{16}$$

(6) 定值边界

$$\text{在 } \Gamma_g \text{ 上：} p_g = \hat{p}_g,\ \text{在 } \Gamma_c \text{ 上：} p_c = \hat{p}_c,\ \text{在 } \Gamma_T \text{ 上：} T = \hat{T},\ \text{在 } \Gamma_u \text{ 上：} u = \hat{u} \tag{17}$$

(7) 定流边界

$$\text{在 } \Gamma_g^q \text{ 上：} (\rho_{ga} v_g - \rho_g v_{gw})\cdot n = q_{ga}$$

$$\text{在 } \Gamma_c^q \text{ 上：} (\rho_{gw} v_g + \rho_w v_w + \rho_g v_{gw})\cdot n = \beta_c(\rho_{gw} - \rho_{gw\infty}) + q_{gw} + q_w$$

$$\text{在 } \Gamma_T^q \text{ 上：} -(\rho_w v_w \Delta h_{vap} - \lambda_{\mathrm{eff}}\,\mathrm{grad}\,T)\cdot n = \alpha_c(T - T_\infty) + q_T \tag{18}$$

其中 n 是孔隙介质表面的单位法向量，T_∞ 和 $\rho_{gw\infty}$ 分别为无穷远处温度和蒸汽密度，q_{ga}，q_{gw}，q_w 和 q_T 分别为边界上的空气流、蒸汽流、液流和热流. 外力边界条件为

$$\text{在 } \Gamma_u^q \text{ 上：} \sigma\cdot n = f \tag{19}$$

1.3 空间与时间离散

对控制方程用 Galerkin 法做空间离散得到如下系统方程

$$-\int_{\Omega} B^{T}\sigma \mathrm{d}\Omega + K_{ug}p_g + K_{uc}p_c + K_{ut}T + F_u = 0 \tag{20}$$

$$C_{gg}\dot{p}_g + C_{gc}\dot{p}_c + C_{gt}\dot{T} + C_{gu}\dot{u} + K_{gg}p_g - K_{gc}p_c + K_{gt}T + F_g = 0,$$

$$C_{cg}\dot{p}_g + C_{cc}\dot{p}_c + C_{ct}\dot{T} + C_{cu}\dot{u} + K_{cg}p_g + K_{cc}p_c + K_{ct}T + F_c = 0,$$

$$C_{tg}\dot{p}_g + C_{tc}\dot{p}_c + C_{tt}\dot{T} + C_{tu}\dot{u} + K_{tg}p_g + K_{tc}p_c + K_{tt}T + F_t = 0,$$

式中, B 为应变-位移转换阵, Ω 为积分区域. p_g, p_c, T 分别为气体压力,毛细压力和温度的有限元离散值向量.将其简写为下述形式的不对称非线性耦合矩阵方程

$$C(X)\dot{X} + K(X)X + F(X) = 0 \tag{21}$$

其中 $X^{T} = \{p_g, p_c, T, u\}$, $C(X)$, $K(X)$ 和 $F(X)$ 由组装式(20)的相应子矩阵得出.对式(21)做时域上的有限差分离散得,

$$C(X_{n+1})\frac{X_{n+1} - X_n}{\Delta t} + K(X_{n+1})X_{n+1} + F(X_{n+1}) = 0 \tag{22}$$

2 非线性不对称方程的并行波前法

2.1 Newton-Raphson 迭代过程

由于矩阵方程(21)是一组非线性方程,用 Newton-Raphson 方法求解有

$$KK\left(X_{n+1}^{\upsilon}\right)\Delta X_{n+1}^{\upsilon} = FF \tag{23}$$

其中

$$KK\left(X_{n+1}^{\upsilon}\right) = \left[\frac{\partial}{\partial X}C\left(X_{n+1}^{\upsilon}\right)\left(X_{n+1}^{\upsilon} - X_n\right) + C\left(X_{n+1}^{\upsilon}\right) + \frac{\partial}{\partial X}K\left(X_{n+1}^{\upsilon}\right)X_{n+1}^{\upsilon} + K\left(X_{n+1}^{\upsilon}\right) + \frac{\partial}{\partial X}F\left(X_{n+1}^{\upsilon}\right)\right]$$

$$FF = -\left[C\left(X_{n+1}^{\upsilon}\right)\left(X_{n+1}^{\upsilon} - X_n\right) + K\left(X_{n+1}^{\upsilon}\right)X_{n+1}^{\upsilon}\Delta t + F\left(X_{n+1}^{\upsilon}\right)\Delta t\right] \tag{24}$$

υ 是迭代次数,解 X 由下式得到

$$X_{n+1}^{\upsilon+1} = X_{n+1}^{\upsilon} + \Delta X_{n+1}^{\upsilon} \tag{25}$$

2.2 并行波前法

将求解区域分成 m 个互不重叠的子区 $\Omega = \bigcup_{j=1}^{m}\Omega_j$,时间 t_{n+1} 时每个子区控制方程有下述形式

$$\begin{bmatrix} kk_b^i\left(x_b\right)_{n+1}^{\upsilon} & kk_{bi}\left(x_i\right)_{n=1}^{\upsilon} \\ kk_{ib}\left(x_b\right)_{n+1}^{\upsilon} & kk_{ii}\left(x_i\right)_{n+1}^{\upsilon} \end{bmatrix}\begin{Bmatrix} \left(\Delta x_b\right)_{n+1}^{\upsilon} \\ \left(\Delta x_i\right)_{n+1}^{\upsilon} \end{Bmatrix} = \begin{Bmatrix} ff^i\left(x_b\right)_{n+1}^{\upsilon} \\ ff\left(x_i\right)_{n+1}^{\upsilon} \end{Bmatrix} \tag{26}$$

其中 kk_b^i 是第 i 个子区的 kk 相应于界面点部分.对每个子区做静凝聚得出

$$\begin{bmatrix} I & kk_{ii}^{-1}\left(x_i\right)_{n+1}^{\upsilon}\ kk_{ib}\left(x_b\right)_{n+1}^{\upsilon} \\ 0 & kk_{bb}^i\left(x_b\right)_{n+1}^{\upsilon} - kk_{bi}\left(x_i\right)_{n+1}^{\upsilon}\ kk_{ii}^{-1}\left(x_i\right)_{n+1}^{\upsilon}\ kk_{ib}\left(x_b\right)_{n+1}^{\upsilon} \end{bmatrix}\begin{Bmatrix} \left(\Delta x_i\right)_{n+1}^{\upsilon} \\ \left(\Delta x_b\right)_{n+1}^{\upsilon} \end{Bmatrix} =$$

$$\left\{ \begin{array}{l} kk_{ii}^{-1}\left(x_i \right)_{n+1}^{v} \quad ff_i\left(x_i \right)_{n+1}^{v} \\ ff_b^i\left(x_b \right)_{n+1}^{v} - kk_{bi}\left(x_i \right)_{n+1}^{v} \quad kk_{ii}^{-1}\left(x_i \right)_{n+1}^{v} \quad ff_i\left(x_i \right)_{n+1}^{v} \end{array} \right\} \tag{27}$$

将静凝聚后每个子区相应于界面点部分组集为总体界面方程

$$\left[\sum_{i=1}^{n} kk_{bb}^{i}\left(x_b \right)_{n+1}^{v} - \sum_{i=1}^{n} kk_{bi}\left(x_i \right)_{n+1}^{v} \quad kk_{ii}^{-1}\left(x_i \right)_{n+1}^{v} \quad kk_{ib}\left(x_b \right)_{n+1}^{v} \right] \left\{ \left(\Delta x_b \right)_{n+1}^{v} \right\} =$$

$$\sum_{i=1}^{n} ff_b^i\left(x_b \right)_{n+1}^{v} - \sum_{i=1}^{n} kk_{bi}\left(x_i \right)_{n+1}^{v} \quad kk_{ii}^{-1}\left(x_i \right)_{n+1}^{v} \quad ff_i\left(x_i \right)_{n+1}^{v} \tag{28}$$

求解界面方程(28)得到 $\left(\Delta x_b \right)_{n+1}^{v}$,再将其回代到式(27)的第一个方程就可得出各子区内部点的值

$$\left(\Delta x_i \right)_{n+1}^{v} = kk_{ii}^{-1}\left(x_i \right)_{n+1}^{v} \left[ff_i\left(x_i \right)_{n+1}^{v} - kk_{ib}\left(x_b \right)_{n+1}^{v} \left(x_b \right)_{n+1}^{v} \right] \tag{29}$$

由于静凝聚和回代都是在各子区内独立进行,可以组织成并行波前过程. 波前过程集子域方程的组集与消元于一体,并不显含式(27～29)中的逆阵运算. 各子区中并行的波前仅含有即将组集的新单元和已组集但尚未消元的单元节点量,它们位于内存一个称为‘活动变量’的数组中. 其操作为

$$\left. \begin{array}{l} kk_{ij}\left(x_k \right)_{n+1}^{v} = kk_{ij}\left(x_k \right)_{n+1}^{v} - \sum \dfrac{kk_{il}\left(x_k \right)_{n+1}^{v} \quad kk_{lj}\left(x_k \right)_{n+1}^{v}}{kk_{ll}\left(x_k \right)_{n+1}^{v}} \\ ff_i\left(x_k \right)_{n+1}^{v} = ff_i\left(x_k \right)_{n+1}^{v} - \sum \dfrac{kk_{il}\left(x_k \right)_{n+1}^{v} \quad ff_i\left(x_k \right)_{n+1}^{v}}{kk_{ll}\left(x_k \right)_{n+1}^{v}} \end{array} \right\} \tag{30}$$

当子区波前达到其边界时,静凝聚过程便完成了,相应于界面方程的量位于各子区的‘活动变量’数组中. 组集并求解界面方程,再在各子区内并行执行波前回代便可得到全部解.

本文用 Master-Slave 并行编程模式在曙光 1000A 上组织上述操作程序. Master 程序负责初始化、向节点处理机播散 Slave 程序和数据、组集与求解界面方程以及收集和显示结果; Slave 程序接收来自 Master 程序的数据、子区的波前运算和向 Master 程序传送结果. Master 与 Slave 程序间的通讯由 PVM 系统提供的通讯函数语句实现. 显然,主要计算由 Slave 程序承担.

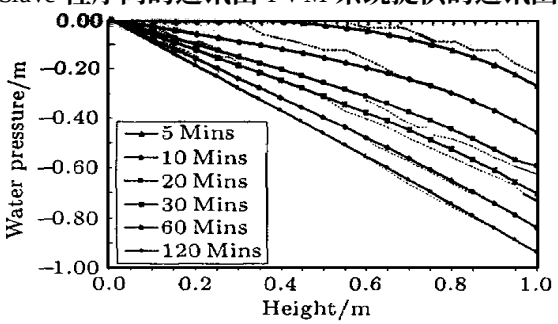

图 1 水压数值解(实线)和实验(文献[9]:虚线)的比较

Fig. 1 Comparison of numerical (solid curves) and experimental results (Ref [9]: dot curves) for water pressure

3 算 例

3.1 Liakopoulos 排水实验[9]

一个高 1 m 的土柱,水自柱顶注入,由底部自由排放,当达到均匀流 ($t = 0$) 时,停止注水并开始实验. 土的孔隙率 $\phi = 29.75\%$. 假定当饱和度 $S \geq 0.91$ 时,饱和度、毛细压力 (p_c) 和相对渗透性 (K_{rl}) 之间关系为

$$S = 1 - 1.9722 \times 10^{-11} P_C^{2.4279}$$

$$K_{rl} = 1 - 2.207(1 - S)^{1.0121}$$

计算时将柱分成 20 个相同尺寸的 8 节点等参

元.在柱两侧边: $q_T = 0$,水平位移 $u_h = 0$;柱顶: $p_g = p_{atm}$ ($p_{atm} = 101325$ Pa) $T = 293.15$ K;柱底: $p_g = p_{atm}$,当 $t > 150$ s 时 $p_c = 0$, $T = 293.15$ K,竖直位移 $u_v = u_h = 0$.土的弹性模量 $E = 1.3$ MPa 泊松比 $\upsilon = 0.4$.在非饱和区假定气压为 p_{atm} .

计算结果的比较见图 1～图 3 ,表 1 , 2 分别给出计算至 300 个和 3600 个时间步时的并行加速比和效率.

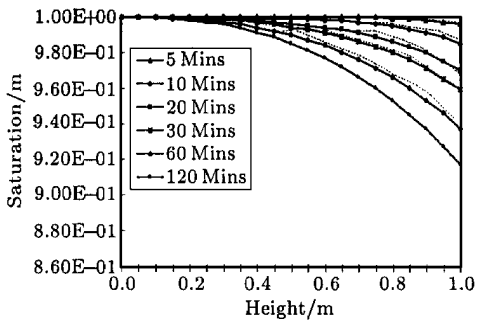

图 2 饱和度数值解(实线)和文献[7] (虚线)的比较

Fig. 2 Resulting saturation profiles (solid curves) compared with solution of Ref [7] (dot curves)

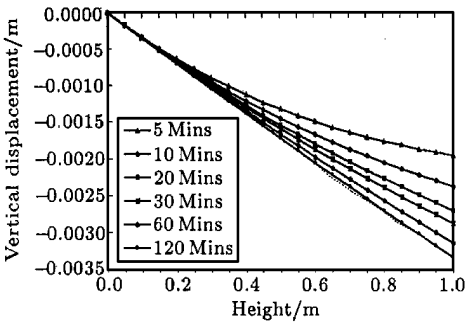

图 3 竖向位移数值解(实线)和文献[7] (虚线)的比较

Fig. 3 Resulting profiles of vertical displacement (solid curves) compared with Ref [7] (dot curves)

表 1　例 1:300 个时间步的加速比和效率

Table 1 Example 1:300 time-steps: Speedup and efficiency

Nodes	D-b CPU time (s)	Speedup	Efficiency (%)	M-f CPU time (s)	Speedup	Efficiency (%)
1	653.91	-	-	76.93	-	-
2	338.81	1.93	96.49	40.21	1.91	95.66
4	230.26	2.84	70.98	28.28	2.72	68.01
5	187.85	3.48	69.62	23.38	3.29	65.81

表 2　例 1:3600 个时间步的加速比和效率

Table 2　Example 1:3600 time-steps: Speedup and efficienc

Nodes	D-b CPU time (s)	Speedup	Efficiency (%)	M-f CPU time (s)	Speedup	Efficiency (%)
1	18223.56	-	-	2106.77	-	-
2	9283.53	1.96	98.15	1088.51	1.94	96.77
4	6384.82	2.85	71.36	754.90	2.79	69.77
5	5144.99	3.54	70.83	626.45	3.36	67.26

3.2　不等温固结[7,8]

这是一个完全饱和的热弹性固结例子.土柱高 7 m 宽 2 m ,弹性模量 $E = 6$ MPa ,泊松比 $\upsilon = 1000$ Pa ,表面温度由初始的 293.15 K.升高 50 K,其他数据与文献[8]相同.计算时将柱分成 18 个 8 节点等参元.柱侧面: $q_T = 0$; $u_h = 0$. 柱顶: $T = 343.15$ K, $p_g = p_{atm}$, $p_c = 0$;柱底: $q_T = 0$, $u_h = 0$. 对开始的 10 个时间步 ,步长为 0.01 天 ,以后每进行 10 个时间步 ,步长扩大 10 倍.

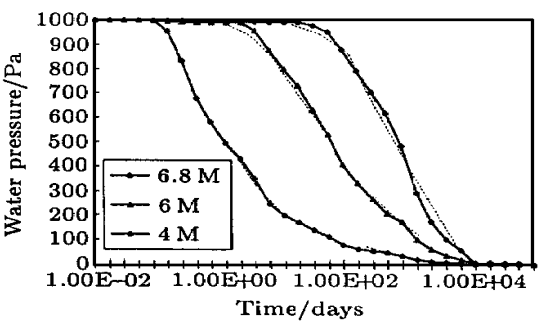

图 4 水压数值解(实线)和文献[7] (虚线)的比较

Fig. 4 Resulting profiles of water pressure (solid curves) compared with Ref [7] (dot curves)

计算结果比较见图 4～图 6,表 3 给出并行加速比和效率. 上述各表中 D-b 表示一般基于带宽格式的并行区域分裂法, M-f 表示并行波前法.

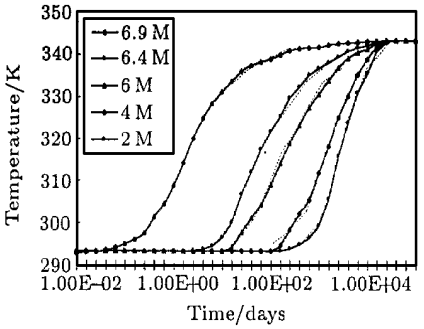

图 5 温度数值解(实线)与文献[7](虚线)的比较

Fig. 5 Resulting profiles of temperature (solid curves) compared with Ref [7] (dot curves)

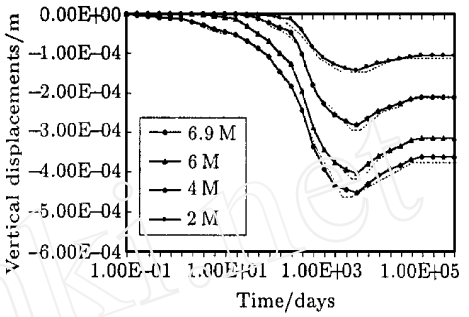

图 6 竖向位移数值解(实线)与文献[7](虚线)的比较

Fig. 6 Resulting profiles of vertical displacement (solid curves) compared with Ref [7] (dot curves)

表 3 例 2:加速比和效率

Table 3 Example 2: Speedup and efficiency

Nodes	D-b CPU	time (s)	Speedup	Efficiency (%)	M-f CPU	time (s)	Speedup	Efficiency (%)
1	260.24	-	-		30.19	-	-	
2	134.84	1.93	96.50		15.83	1.91	95.36	
3	100.79	2.58	86.07		11.87	2.54	84.78	
6	73.94	3.52	58.66		9.53	3.17	52.80	

4 结 论

多相流非线性分析是一个耗时巨大的问题,并行计算是其发展方向. 本文提出的非线性分析并行波前法是求解这类问题的一种有效算法. 它充分利用了波前法可任意编写节点号、占用内存少和计算效率高的优点,又发挥了区域分裂法适于并行计算的特点. 并行程序是采用 Master-Slave 程编模式借助于 PVM 软件包研制的,已经在曙光天潮 1000A 超级计算机上实现. 算例表明,本文的方法不仅有较高的并行加速比和效率,而且其计算时间也远小于基于带宽格式的一般区域分裂法. 由于 Master-Slave 程编模式已广泛地为一般并行计算系统所接受,因而本文的并行程序组织策略有广泛的适用性.

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A PARALELL ANALYSIS METHOD FOR FULL COUPLED MULTIPHACE FLOW[1)]

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Abstract In this paper, we consider a full coupled multi-phase problem involving heat and mass transfer in deforming porous media. The mathematical model consists of balance equations of mass, linear momentum and energy and of the appropriate constitutive equations. The chosen macroscopic field variables are temperature, capillary pressure, gas pressure and displacement. The gas phase is considered to be an ideal gas composed of dry air and vapour, which are regarded as two miscible species. The model makes further use of a modified effective stress concept together with the capillary pressure relationship. Phase change is taken into account as well as heat transfer though conduction and convection and latent heat transfer (evaporation-condensation). Discretization of the non-linear governing equations is carried out by means of finite elements in space and finite differeces in time. A multi-frontal parallel method in conjunction with a Newton-Raphson procedure is developed to solve above problem. The given domain of the problem is descretized into a finite number of subregions or subdomains. Multi-fronts are used to assemble and eliminate internal variable concurrently in every subregion. The contributions for interface equations are obtained from the frontal operating arrays when every wavefront comes to the boundary of its own subregion. Interface equations are solved to obtain the values of the boundary nodes of the subregion. Once the values of the boundary nodes have been determined, the values within each subregion may be determined by a back-substitution routines of the multi-frontal procedures independently.

This method has advantages shch as numbering of the finite element mesh in an arbitrary

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manner, simple programming organisation, smaller core requirements and shorten computation times. The parallel program is developed on Dawning Tiangchao (1000A) parallel computer. The PVM (Parallel Virtual Machine) system is used to handle communications among processors. Numerical examples are given to demonstrated the speedup and efficiency of this method.

Key words parallel algorithm, multiphase flow, deforming porous, phase change