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含双圆孔平板弹性波散射与动应力分析[1)]

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摘要 利用复变量及局部坐标系方法,对平板中含双圆孔弹性波的衍射与动应力集中问题进行了研究,给出了不同方向入射弯曲波条件下该问题一般解的函数逼近序列和边界条件的表达式. 用正交函数展开的方法将待解的问题归结为对一组无穷代数方程组的求解. 给出了双圆孔附近的动应力集中系数的数值结果,分析了孔间距对动应力分布的影响.

关键词 平板, 动应力集中, 复变量方法, 局部坐标系, 双圆孔

引 言

弹性平板结构在航空、航天、船舶及土木建筑工程中经常遇到,工程设计往往要求在平板上开排孔. 由于孔洞间的相互影响,在弹性平板中传播的弯曲波将会发生更复杂的衍射现象,导致开孔附近复杂的动应力分布. 平板开孔弹性波的散射与动应力集中,它直接关系到工程中平板结构的使用寿命[1~5].

N. I. Muskhelishvili 等人提出的复变函数方法已经成功地解决了双调和方程所描述的平板开孔的静应力集中问题[1,2]. 但是,对于平板开孔的动应力集中问题,由于平板弯曲波动方程复杂,用算子因式分解理论将其降阶后得到的是两个 Helmholtz 型方程,所以不能用 Muskhelishvili 提出的复变函数方法求解. 文献[9]在求解二维结构含孔洞弹性波散射与动应力集中时所提出的方法,它是求解用 Helmholtz 型方程所描述的力学问题的有效方法. 60 年代, Y. H. Pao 首次研究了平板开单个圆孔弹性波的散射及动应力集中问题,并给出了问题的分析解与数值结果[6,7]. 而对于平板含多个圆孔弹性波的散射及动应力集中问题,由于问题复杂这方面的研究文献还很少.

本文利用复变量与局部坐标系方法,在平板弯曲波动理论的基础上,对平板中含双圆孔附近的动应力集中问题进行了分析研究,给出了不同方向入射弯曲波条件下该问题一般解的函数逼近序列和边界条件的表达式. 用正交函数展开的方法将待解的问题归结为对一组无穷代数方程组的求解. 本文计算了自由边界的双圆孔附近的动应力集中系数的数值结果,分析研究了孔间距对动应力分布的影响.

1 平板弯曲波动方程和它的一般解

平板开孔弹性波散射及动应力集中问题可归结为求解如下弯曲波动控制方程

$$D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2 \partial y^2} + D\frac{\partial^4 w}{\partial y^4} + \rho h\frac{\partial^2 w}{\partial t^2} = q \tag{1}$$

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式中 D 为平板的弯曲刚度, $D = Eh^3/12(1-v^2)$; E, v 分别为平板的弹性模量和泊松比; ρ, h 分别为平板的密度和厚度; t 为时刻; q 为横向荷载,做自由波动,取 $q = 0$.

求该问题的稳态解,设平板弯曲波动方程的稳态解为 $w = \mathrm{Re}[We^{-i\omega t}]$.

$$\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} - k^4 W = 0 \qquad (2)$$

式中 ω 为平板弯曲波动的圆频率; k 为有量纲波数, $k = \left[\frac{\rho h \omega^2}{D}\right]^{1/4}$. 杂的 ,导

在直角坐标系 (x, y) 中, 弹性平板中的广义内力表达式如下

的 是求 七 中间 平板

$$\left.\begin{aligned} M_x &= -D\left(\frac{\partial^2 W}{\partial x^2} + v\frac{\partial^2 W}{\partial y^2}\right), \quad M_y = -D\left(\frac{\partial^2 W}{\partial y^2} + v\frac{\partial^2 W}{\partial x^2}\right), \quad M_{xy} = -D(1-v)\frac{\partial^2 W}{\partial x \partial y} \\ Q_x &= -D\left(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}\right), \quad Q_y = -D\left(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y}\right) \end{aligned}\right\}$$

动 w 应力 了 ,分 孔附 间 y

式中 W_0 为入射波的幅值; $J_n(\cdot)$ 为第一类 Bessel 函数.

平板中每单个圆孔产生的弹性波散射场可描述为

$$W_m^{(s)} = \sum_{n=-\infty}^{+\infty} [A_n^m H_n^{(1)}(kr_m) e^{in\theta_m} + B_n^m K_n(kr_m) e^{in\theta_m}] \quad (m = 1,2) \tag{8}$$

其中, r_m, θ_m 是第 m 个圆孔的局部极坐标中的极径和极角.

这样,平板中每个圆孔附近的总波场应由入射场与各个开孔产生的散射场叠加而成

$$W = W^{(i)} + \sum_{m=1}^{2} W_m^{(s)} \tag{9}$$

3　开孔附近弹性波的散射及动应力集中

在局部映射平面的单位圆 $\eta = e^{i\theta}$ 上,由文献[10]可得开孔边界上广义内力的限制表达式

$$M_\rho = -\frac{2D(1+\nu)}{\omega'(\eta)\overline{\omega'(\eta)}}\frac{\partial^2 W}{\partial\eta\partial\bar{\eta}} - \frac{D(1-\nu)\eta^2}{\overline{\omega'(\eta)}}\frac{\partial}{\partial\eta}\left[\frac{1}{\omega'(\eta)}\frac{\partial W}{\partial\eta}\right] - \frac{D(1-\nu)\bar{\eta}^2}{\omega'(\eta)}\frac{\partial}{\partial\bar{\eta}}\left[\frac{1}{\overline{\omega'(\eta)}}\frac{\partial W}{\partial\bar{\eta}}\right] =$$

$$L_1^m[W] = F_1^m \tag{10a}$$

$$V_\rho = Q_\rho + \frac{1}{|\omega'(\eta)|}\frac{\partial M_{\rho\theta}}{\partial\theta} = -\frac{D(5-\nu)\eta}{4|\omega'(\eta)|}\frac{\partial}{\partial\eta}[\nabla^2 W] - \frac{D(5-\nu)\bar{\eta}}{4|\omega'(\eta)|}\frac{\partial}{\partial\bar{\eta}}[\nabla^2 W] +$$

$$\frac{2D(1-\nu)\eta^2}{|\omega'(\eta)|\overline{\omega'(\eta)}}\frac{\partial}{\partial\eta}\left[\frac{1}{\omega'(\eta)}\frac{\partial W}{\partial\eta}\right] + \frac{2D(1-\nu)\bar{\eta}^2}{|\omega'(\eta)|\omega'(\eta)}\frac{\partial}{\partial\bar{\eta}}\left[\frac{1}{\overline{\omega'(\eta)}}\frac{\partial W}{\partial\bar{\eta}}\right] +$$

$$\frac{D(1-\nu)\eta^3}{|\omega'(\eta)|\overline{\omega'(\eta)}}\frac{\partial^2}{\partial\eta^2}\left[\frac{1}{\omega'(\eta)}\frac{\partial W}{\partial\eta}\right] + \frac{D(1-\nu)\bar{\eta}^3}{|\omega'(\eta)|\omega'(\eta)}\frac{\partial^2}{\partial\bar{\eta}^2}\left[\frac{1}{\overline{\omega'(\eta)}}\frac{\partial W}{\partial\bar{\eta}}\right] +$$

$$\frac{D(1-\nu)\overline{\omega''(\eta)}\eta}{|\omega'(\eta)|[\omega'(\eta)]^2}\frac{\partial}{\partial\eta}\left[\frac{1}{\omega'(\eta)}\frac{\partial W}{\partial\eta}\right] + \frac{D(1-\nu)\omega''(\eta)\bar{\eta}}{|\omega'(\eta)|[\omega'(\eta)]^2}\frac{\partial}{\partial\bar{\eta}}\left[\frac{1}{\overline{\omega'(\eta)}}\frac{\partial W}{\partial\bar{\eta}}\right] = L_2^m[W] = F_2^m \tag{10b}$$

式中 L_1^m, L_2^m $(m = 1,2)$ 为相应的偏微分算子; F_1^m, F_2^m 为孔边上给定的广义内力.

将式(9)代入满足开孔边界条件式(10),可得如下表达式

$$\sum_{j=1}^{4}\left[\sum_{n=-\infty}^{+\infty} E_n^{ij} X_n^j\right] = E^i \quad (i = 1,2,3,4) \tag{11}$$

其中

$$E_n^{11} = \frac{\eta_1^2\omega'(\eta_1)}{\overline{\omega'(\eta_1)}} H_{n-2}^{(1)}(kr_1) e^{i(n-2)\theta_1} - \frac{2(1+\nu)}{1-\nu} H_n^{(1)}(kr_1) e^{in\theta_1} + \frac{\bar{\eta}_1^2\overline{\omega'(\eta_1)}}{\omega'(\eta_1)} H_{n+2}^{(1)}(kr_1) e^{i(n+2)\theta_1}$$

$$E_n^{12} = \frac{\eta_1^2\omega'(\eta_1)}{\overline{\omega'(\eta_1)}} K_{n-2}(kr_1) e^{i(n-2)\theta_1} + \frac{2(1+\nu)}{1-\nu} K_n(kr_1) e^{in\theta_1} + \frac{\bar{\eta}_1^2\overline{\omega'(\eta_1)}}{\omega'(\eta_1)} K_{n+2}(kr_1) e^{i(n+2)\theta_1}$$

$$E_n^{13} = \frac{\eta_1^2\omega'(\eta_1)}{\overline{\omega'(\eta_1)}} H_{n-2}^{(1)}(kr_2) e^{i(n-2)\theta_2} - \frac{2(1+\nu)}{1-\nu} H_n^{(1)}(kr_2) e^{in\theta_2} + \frac{\bar{\eta}_1^2\overline{\omega'(\eta_1)}}{\omega'(\eta_1)} H_{n+2}^{(1)}(kr_2) e^{i(n+2)\theta_2}$$

$$
E_n^{14} = \frac{\eta_1^2 \omega'(\eta_1)}{\overline{\omega'(\eta_1)}} \mathrm{K}_{n-2}(kr_2)\mathrm{e}^{\mathrm{i}(n-2)\theta_2} + \frac{2(1+\nu)}{1-\nu}\mathrm{K}_n(kr_2)\mathrm{e}^{\mathrm{i}n\theta_2} + \frac{\bar{\eta}_1^2\overline{\omega'(\eta_1)}}{\omega'(\eta_1)} K_{n+2}(kr_2)\mathrm{e}^{\mathrm{i}(n+2)\theta_2}
$$

$$
\begin{aligned}
E_n^{21} = {} & \frac{\eta_1^3[\omega'(\eta_1)]^2}{\overline{\omega'(\eta_1)}}\mathrm{H}^{(1)}_{n-3}(kr_1)\,\mathrm{e}^{\mathrm{i}(n-3)\theta_1} + \frac{4\eta_1^2}{k[\overline{\omega'(\eta_1)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& \mathrm{H}^{(1)}_{n-2}(kr_1)\mathrm{e}^{\mathrm{i}(n-2)\theta_1} + \frac{(5-\nu)}{1-\nu}\eta_1\omega'(\eta_1)\mathrm{H}^{(1)}_{n-1}(kr_1)\mathrm{e}^{\mathrm{i}(n-1)\theta_1} - \frac{(5-\nu)}{1-\nu}\bar{\eta}_1\overline{\omega'(\eta_1)}\times \\
& \mathrm{H}^{(1)}_{n+1}(kr_1)\mathrm{e}^{\mathrm{i}(n+1)\theta_1} + \frac{4\bar{\eta}_1^2}{k[\omega'(\eta_1)]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& \mathrm{H}^{(1)}_{n+2}(kr_1)\mathrm{e}^{\mathrm{i}(n+2)\theta_1} - \frac{\bar{\eta}_1^3[\overline{\omega'(\eta_1)}]^2}{\omega'(\eta_1)}\mathrm{H}^{(1)}_{n+3}(kr_1)\,\mathrm{e}^{\mathrm{i}(n+3)\theta_1}
\end{aligned}
$$

$$
\begin{aligned}
E_n^{22} = {} & -\frac{\eta_1^3[\omega'(\eta_1)]^2}{\overline{\omega'(\eta_1)}}\mathrm{K}_{n-3}(kr_1)\mathrm{e}^{\mathrm{i}(n-3)\theta_1} + \frac{4\eta_1^2}{k[\overline{\omega'(\eta_1)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& \mathrm{K}_{n-2}(kr_1)\mathrm{e}^{\mathrm{i}(n-2)\theta_1} + \frac{(5-\nu)}{1-\nu}\eta_1\omega'(\eta_1)\mathrm{K}_{n-1}(kr_1)\mathrm{e}^{\mathrm{i}(n-1)\theta_1} + \frac{(5-\nu)}{1-\nu}\bar{\eta}_1\overline{\omega'(\eta_1)}\times \\
& \mathrm{K}_{n+1}(kr_1)\mathrm{e}^{\mathrm{i}(n+1)\theta_1} + \frac{4\bar{\eta}_1^2}{k[\omega'(\eta_1)]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& \mathrm{K}_{n+2}(kr_1)\mathrm{e}^{\mathrm{i}(n+2)\theta_1} - \frac{\bar{\eta}_1^3[\overline{\omega'(\eta_1)}]^2}{\omega'(\eta_1)}\mathrm{K}_{n+3}(kr_1)\mathrm{e}^{\mathrm{i}(n+3)\theta_1}
\end{aligned}
$$

$$
\begin{aligned}
E_n^{23} = {} & \frac{\eta_1^3[\omega'(\eta_1)]^2}{\overline{\omega'(\eta_1)}}\mathrm{H}^{(1)}_{n-3}(kr_2)\,\mathrm{e}^{\mathrm{i}(n-3)\theta_2} + \frac{4\eta_1^2}{k[\overline{\omega'(\eta_1)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& \mathrm{H}^{(1)}_{n-2}(kr_2)\mathrm{e}^{\mathrm{i}(n-2)\theta_2} + \frac{(5-\nu)}{1-\nu}\eta_1\omega'(\eta_1)\mathrm{H}^{(1)}_{n-1}(kr_2)\mathrm{e}^{\mathrm{i}(n-1)\theta_2} - \frac{(5-\nu)}{1-\nu}\bar{\eta}_1\overline{\omega'(\eta_1)}\times \\
& \mathrm{H}^{(1)}_{n+1}(kr_2)\mathrm{e}^{\mathrm{i}(n+1)\theta_2} + \frac{4\bar{\eta}_1^2}{k[\omega'(\eta_1)]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& \mathrm{H}^{(1)}_{n+2}(kr_2)\mathrm{e}^{\mathrm{i}(n+2)\theta_2} - \frac{\bar{\eta}_1^3[\overline{\omega'(\eta_1)}]^2}{\omega'(\eta_1)}\mathrm{H}^{(1)}_{n+3}(kr_2)\mathrm{e}^{\mathrm{i}(n+3)\theta_2}
\end{aligned}
$$

$$
\begin{aligned}
E_n^{24} = {} & -\frac{\eta_1^3[\omega'(\eta_1)]^2}{\overline{\omega'(\eta_1)}}\mathrm{K}_{n-3}(kr_2)\mathrm{e}^{\mathrm{i}(n-3)\theta_2} + \frac{4\eta_1^2}{k[\overline{\omega'(\eta_1)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& \mathrm{K}_{n-2}(kr_2)\mathrm{e}^{\mathrm{i}(n-2)\theta_2} + \frac{(5-\nu)}{1-\nu}\eta_1\omega'(\eta_1)\,K_{n-1}(kr_2)\mathrm{e}^{\mathrm{i}(n-1)\theta_2} + \frac{(5-\nu)}{1-\nu}\bar{\eta}_1\overline{\omega'(\eta_1)} + \\
& \mathrm{K}_{n+1}(kr_2)\mathrm{e}^{\mathrm{i}(n+1)\theta_2} + \frac{4\bar{\eta}_1^2}{k[\omega'(\eta_1)]^2}\mathrm{Re}\{\overline{\omega'(\eta_1)}[\omega'(\eta_1)+\eta_1\omega''(\eta_1)]\}\times \\
& K_{n+2}(kr_2)\mathrm{e}^{\mathrm{i}(n+2)\theta_2} - \frac{\bar{\eta}_1^3[\overline{\omega'(\eta_1)}]^2}{\omega'(\eta_1)}K_{n+3}(kr_2)\mathrm{e}^{\mathrm{i}(n+3)\theta_2}
\end{aligned}
$$

$$E_n^{31}=\frac{\eta_2^2\omega'(\eta_2)}{\overline{\omega'(\eta_2)}}H_{n-2}^{(1)}(kr_2)e^{i(n-2)\theta_2}-\frac{2(1+\nu)}{1-\nu}H_n^{(1)}(kr_2)e^{in\theta_1}+\frac{\bar\eta_2^2\overline{\omega'(\eta_2)}}{\omega'(\eta_2)}H_{n+2}^{(1)}(kr_2)e^{i(n+2)\theta_2}$$

$$E_n^{32}=\frac{\eta_2^2\omega'(\eta_2)}{\overline{\omega'(\eta_2)}}K_{n-2}(kr_2)e^{i(n-2)\theta_2}+\frac{2(1+\nu)}{1-\nu}K_n(kr_2)e^{in\theta_2}+\frac{\bar\eta_2^2\overline{\omega'(\eta_2)}}{\omega'(\eta_2)}K_{n+2}(kr_2)e^{i(n+2)\theta_2}$$

$$E_n^{33}=\frac{\eta_2^2\omega'(\eta_2)}{\overline{\omega'(\eta_2)}}H_{n-2}^{(1)}(kr_1)e^{i(n-2)\theta_1}-\frac{2(1+\nu)}{1-\nu}H_n^{(1)}(kr_1)e^{in\theta_1}+\frac{\bar\eta_2^2\overline{\omega'(\eta_2)}}{\omega'(\eta_2)}H_{n+2}^{(1)}(kr_1)e^{i(n+2)\theta_1}$$

$$E_n^{34}=\frac{\eta_2^2\omega'(\eta_2)}{\overline{\omega'(\eta_2)}}K_{n-2}(kr_1)e^{i(n-2)\theta_1}+\frac{2(1+\nu)}{1-\nu}K_n(kr_1)e^{in\theta_1}+\frac{\bar\eta_2^2\overline{\omega'(\eta_2)}}{\omega'(\eta_2)}K_{n+2}(kr_1)e^{i(n+2)\theta_1}$$

$$\begin{aligned}E_n^{41}=&\frac{\eta_2^3[\omega'(\eta_2)]^2}{\overline{\omega'(\eta_2)}}H_{n-3}^{(1)}(kr_2)e^{i(n-3)\theta_2}+\frac{4\eta_2^2}{k[\overline{\omega'(\eta_2)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\}\times\\&H_{n-2}^{(1)}(kr_2)e^{i(n-2)\theta_2}+\frac{(5-\nu)}{1-\nu}\eta_2\omega'(\eta_2)H_{n-1}^{(1)}(kr_2)e^{i(n-1)\theta_2}-\frac{(5-\nu)}{1-\nu}\bar\eta_2\overline{\omega'(\eta_2)}\times\\&H_{n+1}^{(1)}(kr_2)e^{i(n+1)\theta_2}+\frac{4\bar\eta_2^2}{k[\omega'(\eta_2)]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\}\times\\&H_{n+2}^{(1)}(kr_2)e^{i(n+2)\theta_2}-\frac{\bar\eta_2^3[\overline{\omega'(\eta_2)}]^2}{\omega'(\eta_2)}H_{n+3}^{(1)}(kr_2)e^{i(n+3)\theta_2}\end{aligned}$$

$$\begin{aligned}E_n^{42}=&-\frac{\eta_2^3[\omega'(\eta_2)]^2}{\overline{\omega'(\eta_2)}}K_{n-3}(kr_2)e^{i(n-3)\theta_2}+\frac{4\eta_2^2}{k[\overline{\omega'(\eta_2)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\}\times\\&K_{n-2}(kr_2)e^{i(n-2)\theta_2}+\frac{(5-\nu)}{1-\nu}\eta_2\omega'(\eta_2)K_{n-1}(kr_2)e^{i(n-1)\theta_2}+\frac{(5-\nu)}{1-\nu}\bar\eta_2\overline{\omega'(\eta_2)}\times\\&K_{n+1}(kr_2)e^{i(n+1)\theta_2}+\frac{4\bar\eta_2^2}{k[\omega'(\eta_2)]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\}\times\\&K_{n+2}(kr_2)e^{i(n+2)\theta_2}-\frac{\bar\eta_2^3[\overline{\omega'(\eta_2)}]^2}{\omega'(\eta_2)}K_{n+3}(kr_2)e^{i(n+3)\theta_2}\end{aligned}$$

$$\begin{aligned}E_n^{43}=&\frac{\eta_2^3[\omega'(\eta_2)]^2}{\overline{\omega'(\eta_2)}}H_{n-3}^{(1)}(kr_1)e^{i(n-3)\theta_1}+\frac{4\eta_2^2}{k[\overline{\omega'(\eta_2)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\}\times\\&H_{n-2}^{(1)}(kr_1)e^{i(n-2)\theta_1}+\frac{(5-\nu)}{1-\nu}\eta_2\omega'(\eta_2)H_{n-1}^{(1)}(kr_1)e^{i(n-1)\theta_2}-\frac{(5-\nu)}{1-\nu}\bar\eta_2\overline{\omega'(\eta_2)}\times\\&H_{n+1}^{(1)}(kr_1)e^{i(n+1)\theta_2}+\frac{4\bar\eta_2^2}{k[\omega'(\eta_2)]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\}\times\\&H_{n+2}^{(1)}(kr_1)e^{i(n+2)\theta_1}-\frac{\bar\eta_2^3[\overline{\omega'(\eta_2)}]^2}{\omega'(\eta_2)}H_{n+3}^{(1)}(kr_1)e^{i(n+3)\theta_1}\end{aligned}$$

$$E_n^{44}=-\frac{\eta_2^3[\omega'(\eta_2)]^2}{\overline{\omega'(\eta_2)}}K_{n-3}(kr_1)e^{i(n-3)\theta_1}+\frac{4\eta_2^2}{k[\overline{\omega'(\eta_2)}]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\}\times$$

$$K_{n-2}(kr_1)e^{i(n-2)\theta_1} + \frac{(5-v)}{1-v}\eta_2\omega'(\eta_2)K_{n-1}(kr_1)e^{i(n-1)\theta_1} + \frac{(5-v)}{1-v}\bar{\eta}_2\overline{\omega'(\eta_2)} \times$$

$$K_{n+1}(kr_1)e^{i(n+1)\theta_1} + \frac{4\bar{\eta}_2^2}{k[\omega'(\eta_2)]^2}\mathrm{Re}\{\overline{\omega'(\eta_2)}[\omega'(\eta_2)+\eta_2\omega''(\eta_2)]\} \times$$

$$K_{n+2}(kr_1)e^{i(n+2)\theta_1} - \frac{\bar{\eta}_2^3[\overline{\omega'(\eta_2)}]^2}{\omega'(\eta_2)}K_{n+3}(kr_1)e^{i(n+3)\theta_1}$$

$$E^1 = -\frac{4f_1^1}{(1-v)k^2} + \left\{\frac{\eta_1^2\omega'(\eta_1)}{\overline{\omega'(\eta_1)}} + \frac{2(1+v)}{1-v} + \frac{\bar{\eta}_1^2\overline{\omega'(\eta_1)}}{\omega'(\eta_1)}\right\} W_0e^{ikr_1\cos(\theta_1-\theta_0)}$$

$$E^2 = \frac{8|\omega'(\eta_1)|f_2^1}{(1-v)k^3} + \left\{\frac{i\eta_1^3[\omega'(\eta_1)]^2}{\overline{\omega'(\eta_1)}} - \frac{(5-v)i}{1-v}\eta_1\omega'(\eta_1) - \frac{(5-v)i}{1-v}\bar{\eta}_1\overline{\omega'(\eta_1)} + \right.$$

$$W_0e^{ikr_1\cos(\theta_1-\theta_0)}$$

$$E^3 = -\frac{4f_1^2}{(1-v)k^2} + \left\{\frac{\eta_2^2\omega'(\eta_2)}{\overline{\omega'(\eta_2)}} + \frac{2(1+v)}{1-v} + \frac{\bar{\eta}_2^2\overline{\omega'(\eta_2)}}{\omega'(\eta_2)}\right\} W_0e^{ikr_2\cos(\theta_2-\theta_0)}$$

$$E^4 = \frac{8|\omega'(\eta_2)|f_2^2}{(1-v)k^3} + \left\{\frac{i\eta_2^3[\omega'(\eta_2)]^2}{\overline{\omega'(\eta_2)}} - \frac{(5-v)i}{1-v}\eta_2\omega'(\eta_2) - \frac{(5-v)i}{1-v}\bar{\eta}_2\overline{\omega'(\eta_2)} + \right.$$

$$W_0e^{ikr_2\cos(\theta_2-\theta_0)}$$

$$r_2 = \sqrt{L_1^2 + 2L_1a\sin\theta_1 + a^2}, \quad \theta_2 = \cos^{-1}\left(\frac{a\cos\theta_1}{r_2}\right)$$

$$r_1 = \sqrt{L_1^2 - 2L_1a\sin\theta_1 + a^2}, \quad \theta_1 = -\cos^{-1}\left(\frac{a\cos\theta_2}{r_1}\right)$$

式中 $X_n^1 = A_n^1, X_n^2 = B_n^1, X_n^3 = A_n^2, X_n^4 = B_n^2$; η_m 为第 m 个孔局部坐标系的复变量.

用 $e^{-is\theta_m}$ ($i \le 2$ 时, $m=1$; $i \ge 3$ 时, $m=2$) 乘以式(11)的两端,并在区间 $(-\pi,\pi)$ 上积分,可得无穷代数方程组如下

$$\sum_{j=1}^{4}\sum_{n=-\infty}^{+\infty}E_{ns}^{ij}X_n^j = E_s^i \quad (i=j=1,2,3,4; \quad n=s=0,\pm1,\pm2\cdots) \tag{12}$$

式中

$$E_{ns}^{ij} = \frac{1}{2\pi}\int_{-\pi}^{\pi}E_n^{ij}e^{-is\theta_m}d\theta_m, \quad E_s^i = \frac{1}{2\pi}\int_{-\pi}^{\pi}E^ie^{-is\theta_m}d\theta_m$$

式(12)即为确定弹性波模式系数 A_n^m, B_n^m 的无穷代数方程组.

自由孔是人们经常研究一种边界条件. 由开孔动应力集中系数的定义:动弯矩集中系数是开孔周边上的环向动弯矩与入射波在入射方向上的弯矩幅值之比,即有如下表达式

$$M_\theta^* = M_\theta / M_0 \tag{13}$$

式中 M_θ^* 为无量纲弯矩,表示动应力集中系数; M_0 为入射弯矩的幅值, $M_0 = DW_0 k^2$.

由式(3)可得双圆孔间开孔周边上的动应力集中系数为

$$M_\theta^* = (1 + \nu) \sum_{m=1}^{2} \mathrm{Re} \left\{ \frac{1}{W_0} \sum_{n=-\infty}^{+\infty} \left[A_n^m \mathrm{H}_n^{(1)}(kr_m) \mathrm{e}^{\mathrm{i}n\theta_m} + B_n^m \mathrm{K}_n(kr_m) \mathrm{e}^{\mathrm{i}n\theta_m} + \right. \right. \tag{14}$$

式(14)即为平板含双圆孔开孔周边动应力集中系数的一般表达式.

4　数值算例

设有一稳态弯曲波沿与 x 轴正向夹角为 θ_0 的方向传播,平板孔边的边值条件为自由边界条件. 在局部坐标系中圆孔映射为单位圆的映射函数可取如下形式

$$\zeta = \omega(\eta) = r_0 \eta \tag{15}$$

式中, $r_0 = a$, a 是圆孔的半径.

将(15)及各弹性波模式系数代入(14)中,经计算可得开孔附近的动应力集中系数(如图1～8所示). 图中计算的圆孔是上下部署的上圆孔,两圆孔中心的连线与 x 轴垂直. 两圆孔中

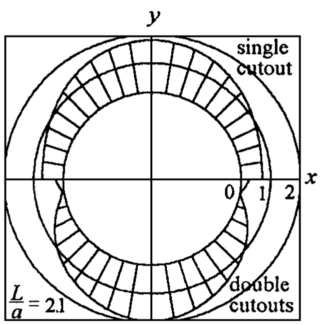

图 1　两孔间孔边动弯矩分布

($ka = 0.2$, $\theta_0 = 0$)

Fig. 1　Dynamic moment factors

($ka = 0.2$, $\theta_0 = 0$)

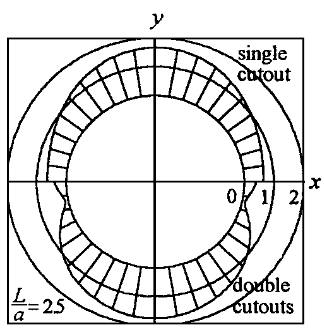

图 2　两孔间孔边动弯矩分布

($ka = 0.5$, $\theta_0 = 0$)

Fig. 2　Dynamic moment factors

($ka = 0.5$, $\theta_0 = 0$)

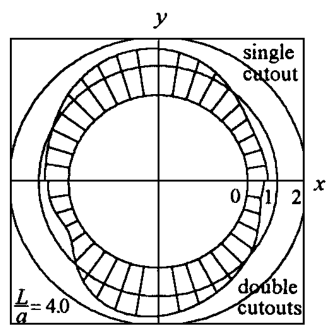

图 3　两孔间孔边动弯矩分布

($ka = 2.0$, $\theta_0 = 0$)

Fig. 3　Dynamic moment factors

($ka = 2.0$, $\theta_0 = 0$)

心的距离为 L. 计算时取 $\nu = 0.30$. 图 1～6 的上半部给出了单圆孔动弯矩集中系数随周向角度(0～ $-\pi$)变化规律. 图 1～6 的下半部则给出了双圆孔间的动弯矩集中系数随周向角度(0～ $-\pi$)变化规律. 图 7 给出了弯曲波同时入射到两开孔($\theta_0 = 0$),双圆孔间的动弯矩集中系数($\theta = -\pi/2$)随无量纲孔间距 L/a 变化的规律. 图 8 则给出了弯曲波先到达下面的开孔,双圆孔间的动弯矩集中系数($\theta = -\pi/2$ 处)随无量纲孔间距 L/a 变化的规律.

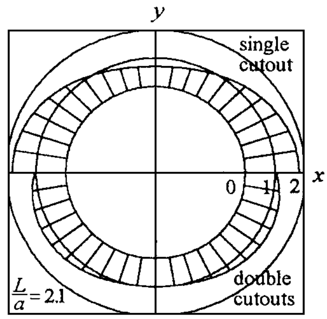

图 4　两孔间孔边动弯矩分布

($ka = 0.2$,　$\theta_0 = \pi/2$)

Fig. 4　Dynamic moment factors

($ka = 0.2$,　$\theta_0 = \pi/2$)

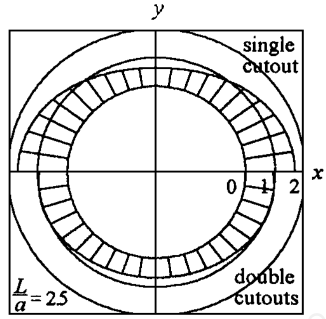

图 5　两孔间孔边动弯矩分布

($ka = 0.5$,　$\theta_0 = \pi/2$)

Fig. 5　Dynamic moment factors

($ka = 0.5$,　$\theta_0 = \pi/2$)

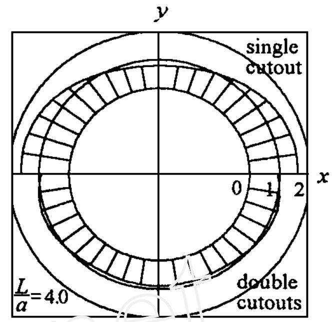

图 6　两孔间孔边动弯矩分布

($ka = 2.0$,　$\theta_0 = \pi/2$)

Fig. 6　Dynamic moment factors

($ka = 2.0$,　$\theta_0 = \pi/2$)

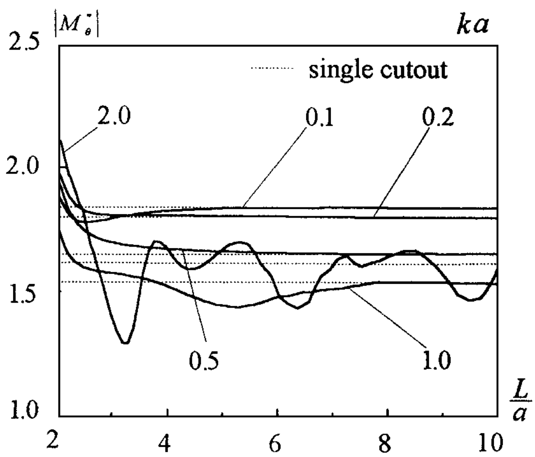

图 7　两孔间动弯矩集中系数随无量纲波数变化规律

($\theta_0 = 0$,　$\theta = -\pi/2$)

Fig. 7　Dynamic moment factors v. s. dimensionless wave number ($\theta_0 = 0$, $\theta = -\pi/2$)

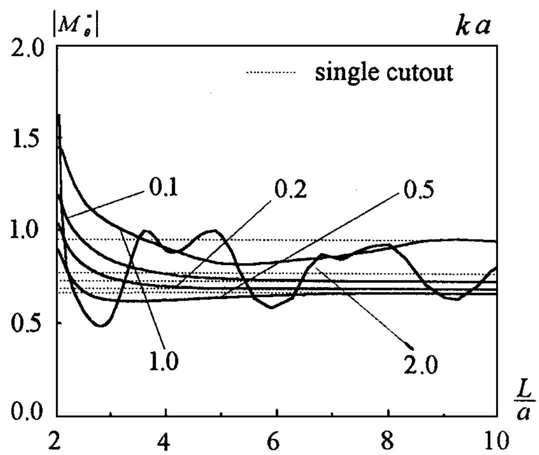

图 8　两孔间动弯矩集中系数随无量纲波数变化规律

($\theta_0 = \pi/2$, $\theta = -\pi/2$)

Fig. 8　Dynamic moment factors v. s. dimensionless wave number ($\theta_0 = \pi/2$,　$\theta = -\pi/2$)

5　结　论

通过对计算结果分析可以看到:

1) 与单圆孔情况相比,由于开孔之间的相互影响,双圆孔间的动应力集中系数要发生比较复杂的变化. 动态应力状态有时会缓解应力集中,但有时也会加剧应力集中. 因此,做动态强度设计时,不能完全套用静载强度设计的标准或规范,最好做一次动态应力分析.

2) 当入射频率比较低时,例如 $ka \leq 0.20$ 时,开孔互不影响节距为 $L/a = 5$;当 $ka \leq 0.5$ 时,开孔互不影响节距为 $L/a = 6$;当 $ka \leq 1.0$ 时,开孔互不影响节距为 $L/a = 9$;当入射频率高时,两孔间距相互影响比较复杂,开孔互不影响节距将变大. 当 $ka \leq 2.0$ 时,开孔互不影响节距为 $L/a = 15$;当 $ka \leq 5.0$ 时,开孔互不影响节距为 $L/a = 25$;当 $ka \leq 8.0$ 时,开孔互不影响节距为 $L/a = 40$. 当孔间动应力集中系数与单孔动应力集中系数之商减去 1 的绝对值小于 2 %时,此时认为开孔互不影响.

3) 与静应力集中问题不同．两孔间距由互不影响距离随着孔间距的减少，孔间的动应力集中系数先是减少而后增加．

本文基于平板弯曲波动理论，利用复变量及局部坐标系方法，对平板中含双圆孔弹性波的衍射与动应力集中问题进行了研究，给出了不同方向入射弯曲波条件下该问题一般解的函数逼近序列和边界条件的表达式．用正交函数展开的方法将待解的问题归结为对一组无穷代数方程组的求解．经典薄板弯曲波动理论局限于低频波动，一般要求 $ka < 8.0$．作为算例，本文给出了双圆孔附近的动应力集中系数的数值结果，并对其进行了讨论．在分析问题时，只要给出如下无量纲量：材料的 Poisson 比 v，无量纲波数 ka，无量纲孔间距 L/a，便可实现对平板含双圆孔的动应力集中问题的分析．可望本文分析及计算结果能对平板结构的动态强度设计起到参考作用．

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DYNAMIC STRESS CONCENTRATIONS IN THIN PLATES WITH TWO CIRCULAR CUTOUTS[1)]

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Abstract The problem of elastic wave motion and dynamic stress concentration in infinite plates , because of its technical importance , has been the subject of many investigators. A number of analytic methods were established for the investigation of stress concentrations , among which , the method developed by N. I. Muskhelishvili is prominent. The problem of static stress concentrations on the edge of an arbitrary cutout can be solved by Muskhelishvili 's method. Scattering of flexural waves and dynamic stress concentrations in plates are different from static stress problem. In 60s , Y. H. Pao investigated scattering of elastic waves and dynamic stress concentrations in thin plates from a circular cavity by using special function method and had given numerical results. In this paper , based on the governing equation for flexural waves of thin plates , diffraction of elastic waves by two cutouts and dynamic stress concentrations in the plate have been studied in terms of the complex variable method and transformation of local coordinates. With reduced order of the governing equations , Helmholtz type equations are obtained. Thus the complex variable method and comformal mapping technique proposed by D. K. Liu can be used for solving diffraction of flexural waves by two cutouts. The general solution of the plate bending problem to satisfy the boundary conditions on the contour of cutouts is obtained. An analytic method to solve dynamic stress concentrations in the plate with two cutouts is established. Using the orthogonal function expansion technique , the dynamic stress problem can be reduced to the solution of a set of infinite algebraic equations. Computational formulas of dynamic stress concentration factors are developed. As examples , numerical results for dynamic moment factors of plates with two circular cutouts are presented under different conditions and some discussions about influence of different parameters , for instance incident waves and structure parameters , on the dynamic stress concentration factors have been made.

Key words thin plate , scattering of elastic wave , dynamic stress concentration , complex variable method and conformal mapping ,local coordinates system , two circular cutouts

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