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复合材料层合扁球壳的非线性强迫振动 1)

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摘要　研究了考虑横向剪切的对称层合圆柱正交异性扁球壳的非线性强迫振动问题 , 得到了共振周期解和非共振周期解. 最后 , 还分析了横向剪切对幅频特性曲线的影响.

关键词　复合材料层合扁球壳 , 横向剪切 , 非线性强迫振动

引　言

由于复合材料优越的特性 , 使得复合材料层合板壳在航空航天等各种工程结构中得到广泛应用. 因此 , 研究复合材料层合板壳的非线性特性 , 大挠度变形 , 非线性屈曲和非线性振动有着十分重要的意义. 在 80 年代 , Chia[1]就对复合材料层合板的后屈曲特性和非线性弯曲振动进行研究. 以后 , 刘人怀[2～9]建立了考虑横向剪切的对称层合圆柱正交异性扁球壳和扁锥壳的大挠度理论以及考虑层间位移和横向剪应力连续条件的层合板理论 , 并对这些板壳的非线性弯曲和稳定问题进行了求解. Wang[10]对复合材料锥壳的有限挠度问题进行了研究. Alwar[11]对复合材料层合环形球壳进行了轴对称的非线性分析. Muc[12]对复合材料层合扁球壳在外压力作用下的屈曲和过屈曲行为进行了分析. Xu[13]对在弹性基础上的对称层合复合材料扁球壳的动态非线性问题进行了研究 , 得到非线性自由振动的 Fourier - bessel 级数解. Qatu 对复合材料层合板壳的非线性剪切变形理论的有效范围进行了讨论.

本文在过去工作的基础上 , 进一步研究了复合材料层合扁球壳的非线性强迫振动问题.

1　非线性强迫振动微分方程

考虑对称层合圆柱正交异性扁球壳 , 受均布强迫荷载 $q(r, t)$ 的作用 , 厚度为 h , 半径为 a , 曲率半径为 R , 拱高为 H.

采用文献 [2] 提出的位移场和修正的剪切刚度 G 的表达式 , 引入动荷载及 Dalenbent 惯性力 , 进行无量纲化 , 得到如下的复合材料层合扁球壳的无量纲非线性强迫振动微分方程

$$\left.\begin{aligned}
&\frac{1}{y}\frac{\partial}{\partial y}[S\Phi + my\Psi + my\Phi - kmy^2] + P - \bar{p}\frac{\partial^2 W}{\partial t^2} = 0\\
&y^{\beta_1}\frac{\partial}{\partial y}y^{1-2\beta_2}\frac{\partial}{\partial y}(y^{\beta_1}\Psi) = my(\Psi + \Phi - ky)\\
&y^{\beta_2}\frac{\partial}{\partial y}y^{1-2\beta_2}\frac{\partial}{\partial y}(y^{\beta_2}S) = -\beta_3(\Phi - ky)\left[\frac{1}{2}(\Phi - ky) + ky\right]
\end{aligned}\right\} \quad (2)$$

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其中

$$
\left.
\begin{aligned}
& y = \frac{r}{a}, \quad W = \frac{w}{h}, \quad \boldsymbol{\Phi} = ky + \frac{\partial w}{\partial y}, \quad \boldsymbol{\Psi} = \frac{a}{h}\boldsymbol{\psi}, \quad S = \frac{a}{D_{11}} rN_r \\
& P = \frac{a^4}{D_{11}h} q(r, t), \quad k = \frac{a^2}{Rh}, \quad m = \frac{a^2}{D_{11}} G, \quad \boldsymbol{\beta}_1^2 = \frac{D_{22}}{D_{11}}, \boldsymbol{\beta}_2^2 = \frac{A_1}{A_3} \\
& \boldsymbol{\beta}_3 = \frac{h^2}{A_3 D^{11}}, \quad \boldsymbol{\beta}_4 = \frac{A_2}{A_3}, \quad \bar{p} = \frac{p}{D_{11}} ha^4 \quad \text{的 有}
\end{aligned}
\right\}
$$

$$H_{10}(y) = my\bar{\rho}[b_1(y^2-1) + b_2(y^3-1) + b_3(y^{4+\beta_1}-1) + b_4(y^5-1)]$$

$$H_{11}(y) = my[H_6(y) - H_6(1)], \quad H_{12}(y) = my[H_7(y) - H_7(1)]$$

$$H_{13}(y) = my[H_8(y) - H_8(1)], \quad H_{14}(y) = a_0(y^3 - y^{\beta_1}) - \frac{y}{m}$$

$$F_1 = a_1 y^7 + a_2 y^{4+\beta_1} + a_3 y^{1+2\beta_1} + a_4 y^5 + a_5 y^{1+\beta_1} + a_6 y^3$$

$$F_2 = a_7 y^5 + a_8 y^{2+\beta_1} + a_9 y^3$$

$$F_i'(y) = \frac{\mathrm{d}F_i}{\mathrm{d}y}, \quad \bar{F}_i(y) = \frac{F_i(y)}{y} \qquad (i = 1,2)$$

$$a_0 = \frac{1}{9-\beta_1^2}, \quad a_1 = -\frac{\beta_3 a_0^2}{2(49-\beta_2^2)}, \quad a_2 = \frac{\beta_3 a_0^2}{(4+\beta_1)^2 - \beta_2^2}$$

$$a_3 = -\frac{\beta_3 a_0^2}{2[(1+2\beta_1)^2 - \beta_2^2]}, \quad a_4 = \frac{\beta_3 a_0}{m(25-\beta_2^2)}, \quad a_5 = -\frac{\beta_3 a_0}{m[(2+\beta_1)^2 - \beta_2^2]}$$

$$a_6 = -\frac{\beta_3}{2m^2(9-\beta_2^2)}, \quad a_7 = \frac{\beta_3 k a_0}{(25-\beta_2^2)}, \quad a_8 = -\frac{\beta_3 k a_0}{(2+\beta_1)^2 - \beta_2}$$

$$a_9 = -\frac{\beta_3 k}{m(9-\beta_2^2)}, \quad a_{10} = \frac{[F_1'(1) - \beta_4 \bar{F}_1(1)]}{\beta_4 - \beta_2}, \quad a_{11} = \frac{[F_i^1(1) - \beta_4 \bar{F}_2(1)]}{\beta_4 - \beta_2}$$

$$b_1 = \frac{a_0}{24(49-\beta_1^2)}, \quad b_2 = -\frac{a_0}{8(9-\beta_1^2)} + \frac{a_0}{2(\beta_1+1)9-\beta_1^2)} + \frac{1}{4m(9-\beta_1^2)}$$

$$b_3 = -\frac{a_0}{(\beta_1+1)(3+\beta_1)[(4+\beta_1)^2 + \beta_3^2]}, \quad b_= - \frac{1}{4m(25-\beta_1^2)} \tag{7}$$

由方程（6）求得壳体关于 $f(t)$ 的具有二次和三次非线性的强迫振动微分方程

$$\frac{\mathrm{d}^2 f(t)}{\mathrm{d}t^2} + \omega_0^2 f(t) + \varepsilon[\lambda f^2(t) + f^3(t)] + b\cos\Omega t = 0 \tag{8}$$

其中

$$\left.\begin{aligned}
&\omega_0^2 = \frac{c_4}{c_1}, \quad \varepsilon = \frac{c_2}{c_1}, \quad \lambda = \frac{c_3}{c_2}, \quad P = q_0\cos\Omega t, \quad b = \frac{c_5}{c_1}q^0 \\
&c_1 = \int_0^1 W_1(y)y\{-\int_0 \bar{\rho} y W_1(y)\mathrm{d}y + H_{10}(y)\}\mathrm{d}y \\
&c_2 = \int_0^1 W_1(y)y[H_1(y)H_{14}(y) - H_{11}(y)]\mathrm{d}y \\
&c_3 = \int_0^1 W_1(y)y[H_2(t)H_{14}(y) + kyH_1(y) - H_{12}(y)]\mathrm{d}y \\
&c_4 = \int_0^1 W_1(y)y[kyH_2(y) - H_{13}(y) + myH_{14}(y)]\mathrm{d}y \\
&c_5 = \int_0^1 W_1(y)y[-H_9(y) + \int y\mathrm{d}y]\mathrm{d}y
\end{aligned}\right\}$$

当 ω_0 与 Ω 既不接近，又不能公约时，我们得到方程（8）的非共振周期解如下

$$
\begin{aligned}
f(t) = {} & \frac{b}{\omega_0^2-\Omega^2}\cos\Omega t+\varepsilon\Big[-\frac{\lambda}{2\omega_0^2}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^2-\frac{3b^3}{4(\omega_0^2-\Omega^2)^4}\cos\Omega t- \\
& \frac{\lambda}{2(\omega_0^2-4\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^2\cos2\Omega t-\frac{1}{4(\omega_0^2-9\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^3\cos3\Omega t\Big]+ \\
& \omega^2\Big\{\frac{1}{\omega_0^2}\Big[\frac{3\lambda}{2\omega_0^2}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^4+\frac{3\lambda}{8(\omega_0^2-4\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^4\Big]+ \\
& \frac{1}{\omega_0^2-\Omega^2}\Big[\frac{\lambda^2}{\omega_0^2}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^3+\frac{\lambda^2}{2(\omega_0^2-4\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^3+ \\
& \frac{15}{8(\omega_0^2-9\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^5\Big]\cos\Omega t+\frac{1}{(\omega_0^2-4\Omega^2)}\Big[\frac{3\lambda}{2\omega_0^2}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^4+ \\
& \frac{\lambda}{\omega_0^2-4\Omega^2}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^4+\frac{3}{8(\omega_0^2-9\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^5\Big]\cos2\Omega t+ \\
& \frac{1}{\omega_0^2-9\Omega^2}\Big[\frac{\lambda^2}{2(\omega_0^2-4\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^3+\frac{9}{16(\omega_0^2-\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^5\Big]\cos3\Omega t+ \\
& \frac{1}{\omega_0^2-16\Omega^2}\Big[\frac{\lambda}{4(\omega_0^2-9\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^4+\frac{3\lambda}{8(\omega_0^2-4\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^4\Big]\cos4\Omega t+ \\
& \frac{1}{\omega_0^2-25\Omega^2}\frac{1}{16(\omega_0^2-9\Omega^2)}\Big(\frac{b}{\omega_0^2-\Omega^2}\Big)^5\cos5\Omega t+O(\varepsilon^3)
\end{aligned}
\tag{10}
$$

当

$$\omega_0^2=\Omega^2+\varepsilon\sigma \tag{11}$$

$$b=\varepsilon M \tag{12}$$

时，我们得到如下共振周期解

$$
\begin{aligned}
f(t) = {} & A_0\cos\Omega t+\varepsilon\Big(A_1\cos\Omega t+\frac{\lambda A_0^2}{6\Omega^2}\cos2\Omega t+\frac{A_0^3}{32}\cos3\Omega t-\frac{\lambda A_0^2}{2\Omega^2}\Big)+ \\
& \varepsilon^2\Big[A_2\cos\Omega t+\Big(\frac{\lambda\sigma}{2\Omega^4}A_0^2+2\lambda A_0A_1+\frac{15\lambda A_0^4}{12\Omega^4}\Big)+\frac{1}{3\Omega^2}\Big(\frac{\lambda\sigma A_0^2}{6\Omega^2}+ \\
& 2\lambda A_0A_1+\frac{15\lambda A_0^4}{32\Omega^2}\Big)\Big]\cos2\Omega t+\frac{1}{8\Omega^2}\Big(\frac{\sigma A_0^3}{32\Omega^2}+\frac{3}{8}A_0^2A_1+ \\
& \frac{\lambda^2A_0^3}{6\Omega^2}+\frac{3A_0^5}{64\Omega^2}\Big)\cos3\Omega t+\frac{1}{15\Omega^2}\Big(\frac{5A_0^4}{32\Omega^2}\Big)\cos4\Omega t+ \\
& \frac{1}{24\Omega^2}\Big(\frac{3A_0^5}{128\Omega^2}\Big)\cos5\Omega t\Big]+O(\varepsilon^3)
\end{aligned}
\tag{13}
$$

这里，A_0 和 A_1 分别由下列方程确定

$$\sigma A_0+\frac{3}{4}A_0^3-M=0 \tag{14}$$

$$A_1=\Big(\frac{5}{6}\lambda^2A_0^3-\frac{3}{4}\frac{A_0^3}{32\Omega^2}\Big)\frac{1}{\sigma+\frac{9}{8}A_0^2} \tag{15}$$

3　幅频特性曲线

对于该非自治系统，在非线性主共振时的特性，由幅频特性曲线来体现.

将式 (11) 代入方程 (14)，得到下列共振曲线方程

$$(\omega_0^2 - \Omega^2) A_0 + \frac{3}{4} \varepsilon A_0^3 - b = 0 \tag{16}$$

现在进行实例计算. 为计算简单，且不失一般性，我们假设壳体各层厚度相等，弹性常数相同，且有 $E_\theta / E_r = 1.5$，$\nu = 0.2$. 于是，由方程 (16)，我们得到如图 1 和图 2 所示的幅频特性曲线.

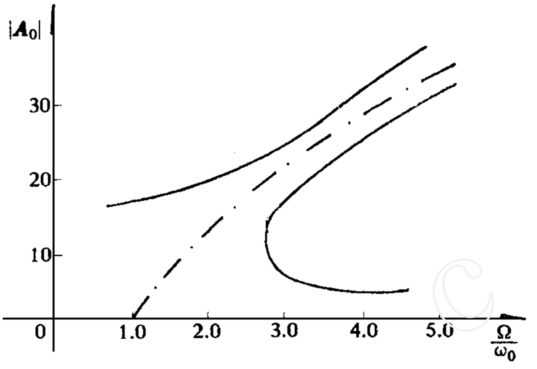

图 1
Fig.1
$P = 15,\ k = 1.4,\ m = 80,\ \bar{\rho} = 1.2$

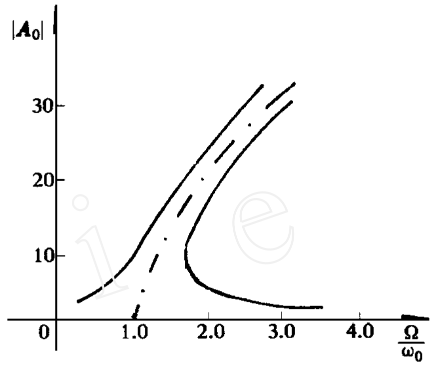

图 2
Fig.2
$P = 5,\ k = 1.4,\ m = 80,\ \bar{\rho} = 1.2$

图 3 和图 4 是在其他参数不变，改变剪切刚度时所得到的幅频特性曲线. 由图看到，当改变剪切刚度时，幅频特性曲线各点切线的斜率发生变化，随着剪切刚度 m 的增加，曲线上各点切线的斜率变大.

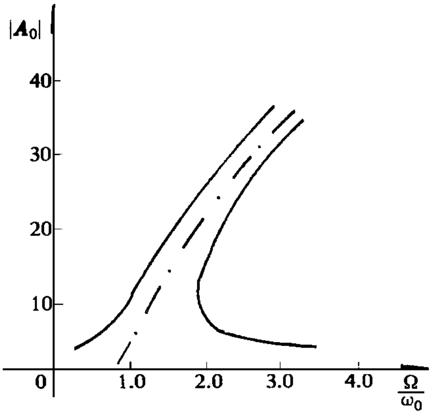

图 3
Fig.3
$m = 800,\ P = 10$

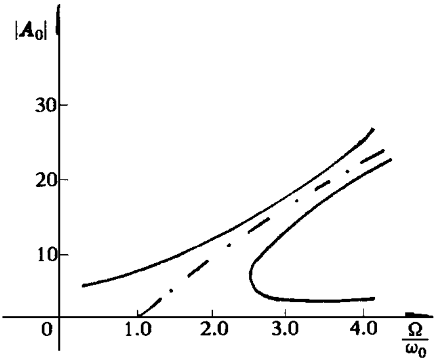

图 4
Fog.4
$m = 8,\ P = 10$

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NONLINEAR FORCED VIBRATION OF LAMINATED COMPOSITE SHALLOW SPHERICAL SHELLS

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Abstract　A nonlinear forcde vibration problem of a symmetrically laminated cylindrically orthotropic shallow spherical shell including transverse shear has been studied. Period solutions of resonance and nonresonance are also obtained. Finally, this paper discusses the effects of transverse shear on characteristic curves of forcing frequency and amplitude of vibration.

Key words　laminated composite shallow spherical shell, transverse shear, nonlinear forced vibration