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某类高阶非完整系方程及高阶WHITTAKER方程

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提要 通过引进的更高阶Euler与Nielsen算子及其性质，给出某类高阶非完整系方程关于广义坐标及准坐标的以更高阶算子表达的共有四种新的等价形式.

给出高阶whittaker方程.

关键词 分析力学；非完整系方程；whittaker方程

一、更高阶Euler与Nielsen算子及某类高阶非完整系方程的等价形式

1. 更高阶Euler与Nielsen算子及性质

把文〔1〕的$k+1$阶约束非完整系关于独立广义坐标与准坐标的共四种Euler与Nielsen算子的微商阶数再提高 i 次，记以$E_1^{(k,i)}$，$N_1^{(k,i)}$，$E_*^{(k,i)}$，$N_*^{(k,i)}$，称为(k,i)阶算子，那末文〔1〕的算子便是$(k,0)$阶，它们的表达形式是相同的，例如$N_1^{(k,i)}(\ )=\partial(\ )^{(k+i+1)}/\partial q_1^{(k+i+1)}-2\partial(\ )^{(k+i)}/\partial q_1^{(k+i)}$. 其中为凝缩起见取广义坐标为矢列$q$，并分块：$q=\begin{pmatrix}q_1\\q_2\end{pmatrix}$，$q_1$为独立广义坐标，$q_*=q_1$为准坐标，矢微商按Jacobi规则进行. $k+1$阶约束系二种等价形式的坐标约束条件微商 i 次并逐次代以约束条件后成为$q_2^{(k+i+1)}=q_2^{(k+i+1)}(q,q^{(1)},\cdots,q^{(k)},q_1^{(k+1)},\cdots,q_1^{(k+i+1)},t)$ 和$q^{(k+i+1)}=q^{(k+i+1)}(q,q^{(1)},\cdots,q^{(k)},q_*^{(k+1)},\cdots,q_*^{(k+i+1)},t)$；同理，动能经高阶微商并代以约束条件后是$\widetilde{T}^{(k+i)}(q,q^{(1)},\cdots,q^{(k)},q_1^{(k+1)},\cdots,q_1^{(k+i+1)},t)=\overset{*}{T}^{(k+i)}(q,q^{(1)},\cdots,q^{(k)},q_*^{(k+1)},\cdots,q_*^{(k+i+1)},t)$.

文〔1〕给出$(k,0)$阶Euler算子与Nielsen算子的关系. 这里得到(k,i)阶Euler算子与Nielsen算子的关系下列（1.1）—（1.4）式，以及(k,i)阶算子与$(k,0)$）阶算子的关系（1.5）—（1.12）式：

$$N_1^{(k,i)}(\widetilde{T})=E_1^{(k,i)}(\widetilde{T}),\quad N_*^{(k,i)}(\overset{*}{T})=E_*^{(k,i)}(\overset{*}{T}),$$
$$N_1^{(k,i)}(q_2^{(1)})=E_1^{(k,i)}(q_2^{(1)}),\quad N_*^{(k,i)}(q^{(1)})=E_*^{(k,i)}(q^{(1)}) \qquad (1.1)—(1.4)$$

$$\begin{pmatrix}N_1^{(k,i)}(\widetilde{T})\\E_1^{(k,i)}(\widetilde{T})\end{pmatrix}-\begin{pmatrix}N_1^{(k,0)}(\widetilde{T})\\E_1^{(k,0)}(\widetilde{T})\end{pmatrix}=-\begin{pmatrix}2\\1\end{pmatrix}(\partial\widetilde{T}^{(k)}/\partial q_2^{(k)})\partial q_2^{(k+1)}/\partial q_1^{(k+1)}$$
$$-i(\partial\widetilde{T}^{(k)}/\partial q_1^{(k+1)})^{(1)}=-\begin{pmatrix}1\\0\end{pmatrix}(\partial\widetilde{T}^{(k)}/\partial q_2^{(k)})\partial q_2^{(k+1)}/\partial q_1^{(k+1)}+\partial\widetilde{T}^{(k)}/\partial q_1^{(k)}$$

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$$-\partial \widetilde{T}^{(k+i)}/\partial q_1^{(k+i)} \qquad (1.5)—(1.6)$$

$$\begin{pmatrix} N_1^{(k,i)}(q_2^{(1)}) \\ E_1^{(k,i)}(q_2^{(1)}) \end{pmatrix} - \begin{pmatrix} N_1^{(k,0)}(q_2^{(1)}) \\ E_1^{(k,0)}(q_2^{(1)}) \end{pmatrix} = -\begin{pmatrix} 2 \\ 1 \end{pmatrix} (\partial q_2^{(k+1)}/\partial q_2^{(k)}) \partial q_2^{(k+1)}/\partial q_1^{(k+1)}$$

$$-i(\partial q_2^{(k+1)}/\partial q_1^{(k+)})^{(1)} = -\begin{pmatrix} 1 \\ 0 \end{pmatrix} (\partial q_2^{(k+1)}/\partial q_2^{(k)}) \partial q_2^{(k+1)}/\partial q_1^{(k+1)}$$

$$+\partial q_2^{(k+1)}/\partial q_1^{(k)} - \partial q_2^{(k+i+1)}/\partial q_1^{(k+i)} \qquad (1.7)—(1.8)$$

$$\begin{pmatrix} N_*^{(k,i)}(\overset{*}{T}) \\ E_*^{(k,i)}(\overset{*}{T}) \end{pmatrix} - \begin{pmatrix} N_*^{(k,0)}(\overset{*}{T}) \\ E_*^{(k,0)}(\overset{*}{T}) \end{pmatrix} = -i(\partial \overset{*}{T}^{(k)}/\partial q_*^{(k+1)})^{(1)}$$

$$= \partial \overset{*}{T}^{(k)}/\partial q_*^{(k)} - \partial \overset{*}{T}^{(k+i)}/\partial q_*^{(k+i)} \qquad (1.9)—(1.10)$$

$$\begin{pmatrix} N_*^{(k,i)}(q^{(1)}) \\ E_*^{(k,i)}(q^{(1)}) \end{pmatrix} - \begin{pmatrix} N_*^{(k,0)}(q^{(1)}) \\ E_*^{(k,0)}(q^{(1)}) \end{pmatrix} = -i(\partial q^{(k+1)}/\partial q_*^{(k+1)})^{(1)}$$

$$= \partial q^{(k+1)}/\partial q_*^{(k)} - \partial q^{(k+i+1)}/\partial q_*^{(k+i)} \qquad (1.11)—(1.12)$$

其中(1.5)、(1.6)式中用括号代表上、下二项，以下也是这样. 只要把$\widetilde{T}^{(k+i)}$，$\overset{*}{T}^{(k+i)}$，$q_2^{(k+i+1)}$，$q^{(k+i+1)}$微商一次，表现为关于依赖的变量的展开形式，即可证明（1.1）—（1.12），这个过程无特殊困难而从略. 其中也得到$i=0$时文〔1〕给出的二种算子的关系式，这个关系式与(1.1)—(1.4) 取$i=0$时的结果比较是：与（1.1）(1.3) 取$i=0$的结果不同，但是与(1.2)(1.4)取$i=0$时的结果相同. 以及注意（1.5）—(1.8) 当$i=0$时是无意义的.

2. 某类高阶非完整系方程的等价形式

成立互为恒等的广义坐标型的(1.13)—(1.17)五式及准坐标型的(1.18)—(1.22)五式：

$$(k+1)\left[\begin{pmatrix} N_1^{(k,0)}(\widetilde{T}) \\ E_1^{(k,0)}(\widetilde{T}) \end{pmatrix} - (\partial T/\partial q_2^{(1)})\begin{pmatrix} N_1^{(k,0)}(q_2^{(1)}) \\ E_1^{(k,0)}(q_2^{(1)}) \end{pmatrix}\right]$$

$$-\begin{pmatrix} k+2 \\ 1 \end{pmatrix}(\partial T^{(k)}/\partial q_2^{(k)})\partial q_2^{(k+1)}/\partial q_1^{(k+1)} + k\,\partial T^{(k)}/\partial q_1^{(k)} - \widetilde{Q}$$

$$= (k+i+1)\left[\begin{pmatrix} N_1^{(k,i)}(\widetilde{T}) \\ E_1^{(k,i)}(\widetilde{T}) \end{pmatrix} - (\partial T/\partial q_2^{(1)})\begin{pmatrix} N_1^{(k,i)}(q_2^{(1)}) \\ E_1^{(k,i)}(q_2^{(1)}) \end{pmatrix}\right]$$

$$+(k+i)\left[(\partial T^{(k+i)}/\partial q_2^{(k+i)})\partial q_2^{(k+1)}/\partial q_1^{(k+1)} + \partial T^{(k+i)}/\partial q_1^{(k+i)}\right] - \widetilde{Q}$$

$$= (\partial \widetilde{T}^{(k)}/\partial q_1^{(k+1)})^{(1)} - \partial T/\partial q_1 - (\partial T/\partial q_2)\partial q_2^{(k+1)}/\partial q_1^{(k+1)}$$

$$-(\partial T/\partial q_2^{(1)})(\partial q_2^{(k+1)}/\partial q_1^{(k+1)})^{(1)} - \widetilde{Q}$$

$$= 0 \qquad (1.13)—(1.17)$$

$$(k+1)\left[\begin{pmatrix} N_*^{(k,0)}(\overset{*}{T}) \\ E_*^{(k,0)}(\overset{*}{T}) \end{pmatrix} - (\partial T/\partial q^{(1)})\begin{pmatrix} N_*^{(k,0)}(q^{(1)}) \\ E_*^{(k,0)}(q^{(1)}) \end{pmatrix}\right]$$

$$+k(\partial T^{(k)}/\partial q^{(k)})\partial q^{(k+1)}/\partial q_*^{(k+1)} - \overset{*}{Q}$$

$$= (k+i+1)\left[\begin{pmatrix} N_*^{(k,i)}(\overset{*}{T}) \\ E_*^{(k,i)}(\overset{*}{T}) \end{pmatrix} - (\partial T/\partial q^{(1)})\begin{pmatrix} N_*^{(k,i)}(q^{(1)}) \\ E_*^{(k,i)}(q^{(1)}) \end{pmatrix}\right]$$

$$+(k+i)(\partial T^{(k+i)}/\partial q^{(k+i)})\,\partial q^{(k+1)}/\partial q_*^{(k+1)} - \overset{*}{Q}$$

$$= (\partial \overset{*}{T}^{(k)}/\partial q_*^{(k+1)})^{(1)} - (\partial T/\partial q)\partial q^{(k+1)}/\partial q_*^{(k+1)} - (\partial T/\partial q^{(1)})$$

$$\cdot(\partial q^{(k+1)}/\partial q_*^{(k+1)})^{(1)} - \overset{*}{Q}$$

$$= 0 \qquad (1.18)—(1.22)$$

文〔1〕给出（1.13)、(1.14)、(1.17）三式的等价性及（1.18)、(1.19)、(1.22）三式的等价性，其中（1.17）及（1.22）分别为文〔2〕得到的 $k+1$ 阶约束系的广义坐标型与准坐标型的Чаплыгин方程．用（k，i）阶算子表达的新的等价形式（1.15)(1.16)(1.20)(1.21）四式是本文得到的．证明这四式的办法是，通过（1.5）－（1.8）由（1.13)(1.14）导出（1.15)(1.16)，以及通过（1.9）－（1.12）由（1.18)(1.19）导出（1.20)(1.21)，这个过程无特殊困难而从略．同样注意，取 $i=0$，由(1.15)(1.16)不能蜕化出（1.13)(1.14)，而（1.20)(1.21）可以蜕化出（1.18)(1.19)．(1.15)(1.16)(1.20)(1.21）四式的物理意义可以用完整系（零阶约束）Mangeron方程[3]进行比拟：完整系提升微分阶数 i 次的等价形式Mangeron方程，相当于（k，0）阶非完整系提升到（k，i）阶等价形式的(1.15)(1.16)(1.20)(1.21)四式．

二、高阶Whittaker方程

1．一种对称守恒性

研究广义方程（1.14）的对称守恒性．(1.14）改写为

$$(k+1)(\partial\widetilde{T}^{(k)}/\partial q_1^{(k+1)})^{(1)}-\partial\widetilde{T}^{(k)}/\partial q_1^{(k)}-\partial T^{(k)}/\partial q_2^{(k+1)}[(k+1)(\partial q_2^{(k+1)}/\partial q_1^{(k+1)})^{(1)}-\partial q_2^{(k+1)}/\partial q_1^{(k)}]+(\partial T^{(k)}/\partial q_2^{(k)})\partial q_2^{(k+1)}/\partial q_1^{(k+1)}-\widetilde{Q}=0 \tag{1.14)'$$

上式乘以 $q_1^{(k+1)}$，在下列条件下

$$(\partial q_2^{(k+1)}/\partial q_1^{(k+1)})q_1^{(k+1)}-q_2^{(k+1)}=0\,,\quad \partial T^{(k)}/\partial t=0 \tag{2.1}$$

转为

$$(k+1)[(\partial\widetilde{T}^{(k)}/\partial q_1^{(k+1)})q_1^{(k+1)}-\widetilde{T}^{(k)}]^{(1)}+(k+1)T^{(k+1)}-(k+1)(\partial T^{(k)}/\partial q^{(k+1)})q^{(k+2)}-(\partial T^{(k)}/\partial q^{(k)})q^{(k+1)}-\widetilde{Q}q_1^{(k+1)}=0 \tag{2.2}$$

因此再加以条件

$$(k+1)T^{(k+1)}-(k+1)(\partial T^{(k)}/\partial q^{(k+1)})q^{(k+2)}-(\partial T^{(k)}/\partial q^{(k)})q^{(k+1)}-\widetilde{Q}q^{(k+1)}=0 \tag{2.3}$$

成立守恒积分

$$(\partial\widetilde{T}^{(k)}/\partial q_1^{(k+1)})q_1^{(k+1)}-\widetilde{T}^{(k)}=\text{常数} \tag{2.4}$$

(2.1)(2.3)(2.4）为本文得到的高阶约束系方程（1.14)′的一种对称守恒性．当 $k=0$，$\widetilde{Q}=0$，则(2.3）恒成立而不需要存在．若再蜕化为 $\widetilde{T}=\widetilde{T}(q_1,q_1^{(1)})$ 即不依赖于 q_2，则为文〔4〕研究的情况．

2．广义Whittaker方程

在条件（2.1)(2.3)，及再加以条件

$$\partial q_2^{(k+1)}/\partial t=0 \tag{2.5}$$

(1.14)′转为

$$\theta^{(1)}(k+1)E_{1\theta}^{(k,0)}(\widetilde{\widetilde{T}})+\theta^{(1)}k\,\partial\widetilde{\widetilde{T}}^{(k)}/\partial\bar{q}_1^{(k)}-\theta^{(1)}(k+1)\overline{(\partial T^{(k)}/\partial q_2^{(k+1)})}E_{1\theta}^{(k,0)}(\widetilde{q}_2^{(1)})-\theta^{(1)}k\,\overline{(\partial T^{(k)}/\partial q_2^{(k+1)})}\partial q_2^{(k+1)}/\partial\bar{q}_1^{(k)}-\overline{(\partial T^{(k)}/\partial q_2^{(k)})}\cdot\partial\widetilde{q}_2^{(k+1)}/\partial\bar{q}_1^{(k)\prime}-\bar{\widetilde{Q}}=0 \tag{1.14)''$$

其中

$$q_1^{(k)}=\begin{pmatrix}\bar{q}_1^{(k}\\ \theta\end{pmatrix},\bar{q}^{(k)\prime}=d\bar{q}_1^{(k)}/d\theta,\bar{q}_1^{(k+1)}=\bar{q}_1^{(k)\prime}\theta^{(1)} \tag{2.6}$$

(θ 为1 × 1阶)

$$E_{1\theta}^{(k,0)}(\quad)=d[\partial(\quad)^{(k)}/\partial\bar{q}_1^{(k)\prime}]/d\theta-\partial(\quad)^{(k)}/\partial\bar{q}_1^{(k)} \tag{2.7}$$

$\widetilde{\widetilde{T}}^{(k)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime})$及$\tilde{q}_2^{(k+1)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime})$的含义见附录（3.7)(3.12）式，以及(　)为用变量（$q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime}$）表示的(　)值.(1.14)″的证明见附录．（1.14)″为本文得到的高阶非完整系的广义Wattaker方程,代以附录式（3.4)(3.12）后成为变量$\bar{q}_1^{(k)}$关于自变量θ的二阶微分方程，其中由于（q，$q^{(1)},\cdots,q^{(k-1)},q_2^{(k)}$不依赖于 θ 而将作为参量存在，因此（1.14)″的积分形式是$\bar{q}_1^{(k)}=q_1^{(k)}(q,q^{(1)},\cdots,q^{(k-1)},q_2^{(k)},\theta)$．当$k=0$，$\widetilde{Q}=0$，$\widetilde{T}=\widetilde{T}(q_1,q_1^{(1)})$,(1.14)″蜕化为文〔4〕的结果.

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附录　式（1.14)″ 的证明

令

$$\Omega^{(k)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime},\theta^{(1)})=\widetilde{T}^{(k)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime}\theta^{(1)},\theta^{(1)}) \tag{3.1}$$

易证

$$\begin{aligned}\partial\Omega^{(k)}/\partial\theta^{(1)}&=\partial\widetilde{T}^{(k)}/\partial\theta^{(1)}+(\partial\widetilde{T}^{(k)}/\partial\bar{q}_1^{(k+1)})\partial\bar{q}_1^{(k+1)}/\partial\theta^{(1)}\\&=(\partial\widetilde{T}^{(k)}/\partial q_1^{(k+1)})q_1^{(k+1)}/\theta^{(1)}\end{aligned} \tag{3.2}$$

$$\partial\Omega^{(k)}/\partial q_1^{(k)\prime}=\theta^{(1)}\partial\widetilde{T}^{(k)}/\partial\bar{q}_1^{(k+1)},\ \partial\Omega^{(k)}/\partial q_1^{(k)}=\partial\widetilde{T}^{(k)}/\partial q_1^{(k)},$$
$$\partial\Omega^{(k)}/\partial q_2^{(k)}=\partial\widetilde{T}^{(k)}/\partial q_2^{(k)} \tag{3.3}$$

由（2.4)(2.6）解出

$$\theta^{(1)}=\theta^{(1)}(q,q^{(1)},\cdots,q^{(k)},q^{(k)\prime}) \tag{3.4}$$

由（2.4)(3.1)(2.2）得

$$(\partial\Omega^{(k)}/\partial\theta^{(1)})\theta^{(1)}-\Omega^{(k)}=\text{常数} \tag{3.5}$$

上式微分，通过（3.4）得

$$\left.\begin{aligned}&\theta^{(1)}(\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial\theta^{(1)})\partial\theta^{(1)}/\partial\bar{q}_1^{(k)\prime}+\theta^{(1)}\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial\bar{q}_1^{(k)\prime}-\partial\Omega^{(k)}/\partial\bar{q}_1^{(k)\prime}=0\\&\theta^{(1)}(\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial\theta^{(1)})\partial\theta^{(1)}/\partial q_1^{(k)}+\theta^{(1)}\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial q_1^{(k)}-\partial\Omega^{(k)}/\partial q_1^{(k)}=0\\&\theta^{(1)}(\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial\theta^{(1)})\partial\theta^{(1)}/\partial q_2^{(k)}+\theta^{(1)}\partial^2\Omega/\partial\theta^{(1)}\partial q_2^{(k)}-\partial\Omega^{(k)}/\partial q_2^{(k)}=0\end{aligned}\right\} \tag{3.6}$$

令

$$\widetilde{\widetilde{T}}^{(k)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime})=\partial\Omega^{(k)}/\partial\theta^{(1)} \tag{3.7}$$

其中等号左边代以了（3.4)；再进行微分：

$$\left.\begin{aligned}&\partial\widetilde{\widetilde{T}}^{(k)}/\partial\bar{q}_1^{(k)\prime}=(\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial\theta^{(1)})\partial\theta^{(1)}/\partial\bar{q}_1^{(k)\prime}+\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial q_1^{(k)\prime}\\&\partial\widetilde{\widetilde{T}}^{(k)}/\partial q_1^{(k)}=(\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial\theta^{(1)})\partial\theta^{(1)}/\partial q_1^{(k)}+\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial q_1^{(k)}\\&\partial\widetilde{\widetilde{T}}^{(k)}/\partial q_2^{(k)}=(\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial\theta^{(1)})\partial\theta^{(1)}/\partial q_2^{(k)}+\partial^2\Omega^{(k)}/\partial\theta^{(1)}\partial q_2^{(k)}\end{aligned}\right\} \tag{3.8}$$

（3.6）三式与（3.8）三式各别联合，得

$$\partial\widetilde{\widetilde{T}}^{(k)}/\partial\bar{q}_1^{(k)\prime}=\theta^{(1)-1}\partial\Omega^{(k)}/\partial\bar{q}_1^{(k)\prime},\ \partial\widetilde{\widetilde{T}}^{(k)}/\partial q_1^{(k)}=\theta^{(1)-1}\partial\Omega^{(k)}/\partial q_1^{(k)},$$
$$\partial\widetilde{\widetilde{T}}^{(k)}/\partial q_2^{(k)}=\theta^{(1)-1}\partial\Omega^{(k)}/\partial q_2^{(k)} \tag{3.9}$$

（3.3）三式与（3.9）三式各别联合，得

$$\partial\widetilde{\widetilde{T}}^{(k)}/\partial\bar{q}_1^{(k)\prime}=\partial\widetilde{T}^{(k)}/\partial\bar{q}_1^{(k+1)},\ \partial\widetilde{\widetilde{T}}^{(k)}/\partial q_1^{(1)}=\theta^{(1)-1}\partial\widetilde{T}^{(k)}/\partial q_1^{(k)},$$
$$\partial\widetilde{\widetilde{T}}^{(k)}/\partial q_2^{(k)}=\theta^{(1)-1}\partial\widetilde{T}^{(k)}/\partial q_2^{(k)} \tag{3.10}$$

$$\omega_2^{(k)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime},\theta^{(1)})=q_2^{(k+1)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime}\theta^{(1)},\theta^{(1)}) \tag{3.11}$$

$$q_2^{(k+1)}(q,q^{(1)},\cdots,q^{(k)},\bar{q}_1^{(k)\prime})=\partial\omega_2^{(k)}/\partial\theta^{(1)} \tag{3.12}$$

比较 (3.1) 和((3.11)、(3.7) 和 (3.12),(2.1) 和 (2.4)(2.5),用相同步骤导出

$$(\partial\omega_2^{(k)}/\partial\theta^{(1)})\theta^{(1)}-\omega_2^{(k)}=0 \tag{3.13}$$

及与 (3.10) 类似的结果

$$\partial\tilde{q}_2^{(k+1)}/\partial\bar{q}_1^{(k)\prime}=\partial q_2^{(k+1)}/\partial\bar{q}_1^{(k+1)},\quad \partial\tilde{q}_2^{(k+1)}/\partial q_1^{(k)}=\theta^{(1)^{-1}}\partial q_2^{(k+1)}/\partial q_1^{(k)},$$
$$\partial\tilde{q}_2^{(k+1)}/\partial q_2^{(k)}=\theta^{(1)^{-1}}\partial q_2^{(k+1)}/\partial q_2^{(k)} \tag{3.14}$$

(3.10)(3.14) 代入 (1.14)′ 即证得 (1.14)″.

THE GENERALIZED EQUATIONS AND WHITTAKER EQUATIONS OF HIGHER ORDER IN NONHOLONOMIC SYSTEMS

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ABSTRACT In this paper, the new generalized equations and the whittaker equations of nonholonomic systems of higher order are obtained, and the special conditions in paper [1—4] are included.

KEY WORDS Analytical mechanics, Nonholonomic systems of higher order, Whittaker equation.