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大涡模拟的代数模型

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提要 本文提出了一种用于计算拟雷诺应力中各向同性及非均匀二部份的代数模型.将这种代数模型用作大涡模拟中的湍流模型.该模型首先用于直槽道内的湍流运动的数值模拟．初步结果表明该模型是有希望的．

关键词 湍流；大涡模拟；计算流体动力学．

一、引　　言

大涡模拟是近年来发展起来的一种研究湍流的有前途的手段．这一思想最早由Smagorinsky 在1963年提出的．而后由 Deardorff[2] 首先实现，以后 Moin[3]、Schumann[4]、Anonoponlos-Dennis[5] 等都作了不少的改进和尝试，并取得不少的进展．大涡模型，对于壁面附近的情况仍有很多不足之处.本文目的是试图用代数模型来计算子格拟雷诺应力，首先对直槽道内的湍流运动进行计算，并与 Schumann 的结果作比较．以后将把它们推广到弯曲槽道内的湍流数值模拟．

二、出发方程

不可压流体湍流运动的基本方程仍为N-S 方程，它们可以表达为向量的形式

$$\nabla\cdot\boldsymbol{V}=0 \qquad (1)$$

和

$$\frac{D\boldsymbol{V}}{Dt}=-\nabla\left(\frac{p}{\rho}\right)+\mathrm{Div}(2\nu\mathbf{S}) \qquad (2)$$

其中$\mathbf{S}$ 为变形率张量，由上述方程可以导出流体动能方程

$$\frac{DE}{Dt}=-\nabla\cdot\left(\boldsymbol{V}\frac{p}{\rho}\right)+\nabla\cdot(2\nu\mathbf{S}\boldsymbol{V})-2\nu S_{kl}S_{kl} \qquad (3)$$

及速度相关量方程

$$\frac{\partial v_iv_j}{\partial t}+(v_i\boldsymbol{e}_j+v_j\boldsymbol{e}_i)\cdot[(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}]$$

$$=-\nabla\cdot\left[\frac{p}{\rho}(v_i\boldsymbol{e}_j+v_j\boldsymbol{e}_i)\right]+\frac{p}{\rho}\nabla\cdot(v_i\boldsymbol{e}_j+v_j\boldsymbol{e}_i)$$

$$+2\nu\left\{\nabla\cdot\left[\frac{1}{2}(\boldsymbol{e}_j\cdot(\boldsymbol{e}_k\cdot\nabla)+\boldsymbol{e}_k\cdot(\boldsymbol{e}_j\cdot\nabla))(v_i\boldsymbol{V})\right.\right.$$

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$$+\frac{1}{2}(\boldsymbol{e}_i\cdot(\boldsymbol{e}_k\cdot\nabla)+\boldsymbol{e}_k\cdot(\boldsymbol{e}_i\cdot\nabla))(v_i\mathbf{V})$$

$$-\frac{1}{2}\nabla\cdot\left[\left(\frac{1}{h_k}\frac{\partial v_iv_j}{\partial q_k}+\frac{v_k}{h_i}\frac{\partial v_i}{\partial q_j}+\frac{v_k}{h_i}\frac{\partial v_j}{\partial q_i}\right)\cdot{}_k\right]$$

$$-\left[\frac{S_{jk}}{h_k}\frac{\partial}{\partial q_k}\left(\frac{v_i}{h_i}\right)+\frac{S_{ik}h_i}{h_k}\frac{\partial}{\partial q_k}\left(\frac{v_j}{h_j}\right)\right]$$

$$-S_{kk}\left(\frac{v_i}{h_jh_k}\frac{\partial h_k}{\partial q_j}+\frac{v_j}{h_ih_k}\frac{\partial h_k}{\partial q_i}\right)\Bigg\}\tag{4}$$

其中$\boldsymbol{e}_i$、$\boldsymbol{e}_j$、$\boldsymbol{e}_k$分别为单位向量，$h_ih_jh_k$为相应的Lame系数，$q_iq_jq_k$为正交曲线座标系．（1）,（2）方程为出发方程,（3）,（4）为导出方程．

三、大尺度量的基本方程

根据大涡模拟的基本思想，将滤波后的物理量称作大尺度量，大尺度量与原物理量的差别为小尺度量．这里采用的滤波函数$G(x|x')$为

$$G(x|x')=\begin{cases}\dfrac{1}{\Delta_1\Delta_2\Delta_3}, & |x_i-x_i'|<\dfrac{\Delta_i}{2}\quad i=1,2,3\\ 0 & |x_i-x_i'|>\dfrac{\Delta_i}{2}\quad i=1\text{ 或 }2\text{ 或 }3\end{cases}\tag{5}$$

过滤后量为$\bar{f}$，其中

$$\bar{f}=\int_{\Delta V}G\cdot f\,d(\Delta V);\qquad f'=f-\bar{f}\tag{6}$$

ΔV为网格单元体积，这里假定ΔV足够小，在其中h_i的变化可以略去不计，故

$$\overline{g(q_i)\cdot f}=\int_{\Delta V}G\cdot g(q_i)f\,d(\Delta V)\tag{7}$$

中$g(q_i)$表示与q_i有关的几何量，如h_i等．用滤波函数（5）对方程（1）—（4）进行滤波后可得大尺度量的基本方程为

$$\frac{1}{h_k}\frac{\partial\bar{v}_k}{\partial q_k}+\frac{\bar{v}_k}{h_lh_k}\frac{\partial h_l}{\partial q_k}-\frac{\bar{v}_k}{h_k^2}\frac{\partial h_k}{\partial q_k}=0\tag{8}$$

$$\frac{\partial\bar{v}_i}{\partial t}+\frac{\bar{v}_k}{h_k}\frac{\partial\bar{v}_i}{\partial q_k}+\frac{\bar{v}_i\bar{v}_k}{h_ih_k}\frac{\partial h_i}{\partial q_k}-\frac{\bar{v}_k\bar{v}_k}{h_ih_k}\frac{\partial h_k}{\partial q_i}$$

$$=-\frac{1}{h_i}\frac{\partial}{\partial q_i}\left(\frac{\bar{p}}{\rho}\right)+\left\{\frac{1}{h_k}\frac{\partial\tau_{ik}}{\partial q_k}+\frac{\tau_{ik}}{h_lh_k}\frac{\partial h_k}{\partial q_l}-\frac{\tau_{ik}}{h_k^2}\frac{\partial h_k}{\partial q_k}\right.$$

$$\left.-\frac{\tau_{kk}}{h_ih_k}\frac{\partial h_k}{\partial q_i}+\frac{\tau_{ik}}{h_ih_k}\frac{\partial h_i}{\partial q_k}\right\}\tag{9}$$

其中$\tau_{ik}=\overline{\tau_{ik}}+\tau_{ik}^*=2\nu\bar{S}_{ik}+(-\overline{v_i'v_k'})$, $v_i'=v_i-\bar{v}_i$,

$$\frac{\partial\overline{E'}}{\partial t}+\frac{\bar{v}_k}{h_k}\frac{\partial\overline{E'}}{\partial q_k}+\frac{\overline{v_k'v_l'}}{h_k}\frac{\partial\bar{v}_l}{\partial q_k}+\frac{\overline{v_k'}}{h_k}\frac{\partial\overline{E'}}{\partial q_k}-\bar{v}_l\left(\frac{\overline{v_l'v_k'}}{h_lh_k}\frac{\partial h_l}{\partial q_k}-\frac{\overline{v_k'v_k'}}{h_lh_k}\frac{\partial h_k}{\partial q_l}\right)$$

$$
-\bar{v}_l\left(\frac{\bar{v}_l\bar{v}_k}{h_l h_k}\frac{\partial h_l}{\partial q_k}-\frac{\bar{v}_k\bar{v}_k}{h_l h_k}\frac{\partial h_k}{\partial q_l}\right)=-\frac{1}{h_k}\frac{\partial}{\partial q_k}\left(\overline{v_k'\frac{p'}{\rho}}\right)+\overline{\frac{p'}{\rho h_k}\frac{\partial v_k'}{\partial q_k}}
$$

$$
-2\nu\frac{1}{\hbar}\frac{\partial}{\partial q_k}\left(\overline{v_k' S_{kl}'}\ \frac{\hbar}{h_k}\right)-\varepsilon \tag{10}
$$

其中 $\hbar=h_1h_2h_3$，$\varepsilon=2\nu\overline{S_{kl}'S_{kl}'}$；$E'=(1/2)\overline{v_l'v_l'}$，

$$
\frac{\partial\overline{v_i'v_j'}}{\partial t}+\frac{\bar{v}_k}{h_k}\frac{\partial\overline{v_i'v_j'}}{\partial q_k}+\overline{v_i'v_k'}\frac{1}{h_k}\frac{\partial\bar{v}_j}{\partial q_k}+\overline{v_j'v_k'}\frac{1}{h_k}\frac{\partial\bar{v}_i}{\partial q_k}
$$

$$
+\overline{v_i'v_j'}\left(\frac{\bar{v}_k}{h_ih_k}\frac{\partial h_i}{\partial q_k}+\frac{\bar{v}_k}{h_jh_k}\frac{\partial h_j}{\partial q_k}\right)+\overline{v_j'v_k'}\left(\frac{\bar{v}_i}{h_ih_k}\frac{\partial h_i}{\partial q_k}-\frac{2\bar{v}_k}{h_ih_k}\frac{\partial h_k}{\partial q_i}\right)
$$

$$
+\overline{v_i'v_k'}\left(\frac{\bar{v}_j}{h_jh_k}\frac{\partial h_j}{\partial q_k}-\frac{2\bar{v}_k}{h_jh_k}\frac{\partial h_k}{\partial q_j}\right)+\frac{1}{h_k}\frac{\partial}{\partial q_k}\overline{v_i'v_j'v_k'}+\overline{v_i'v_j'v_k'}\left(\frac{1}{h_kh_l}\frac{\partial h_l}{\partial q_k}\right.
$$

$$
\left.-\frac{1}{h_k^2}\frac{\partial h_k}{\partial q_k}\right)-\left(\frac{\overline{v_k'v_k'v_j'}}{h_ih_k}\frac{\partial h_k}{\partial q_i}-\frac{\overline{v_k'v_k'v_i'}}{h_jh_k}\frac{\partial h_k}{\partial q_j}\right)
$$

$$
=-\left\{\frac{1}{h_i}\frac{\partial}{\partial q_i}\left(\overline{v_j'\frac{p'}{\rho}}\right)+\frac{1}{h_j}\frac{\partial}{\partial q_j}\left(\overline{v_i'\frac{p'}{\rho}}\right)\right\}+\overline{\frac{p'}{\rho}\left(\frac{1}{h_i}\frac{\partial v_j'}{\partial q_i}+\frac{1}{h_j}\frac{\partial v_i'}{\partial q_j}\right)}+\nu\left\{\Delta\overline{v_i'v_j'}\right.
$$

$$
+\nabla\cdot\left[\left(\overline{v_i'\frac{h_k}{h_j}\frac{\partial}{\partial q_j}\left(\frac{v_j'}{h_k}\right)}+\overline{v_j'\frac{h_k}{h_i}\frac{\partial}{\partial q_i}\left(\frac{v_i'}{h_k}\right)}-\frac{\overline{v_i'v_j'}}{h_ih_jh_k}\frac{\partial h_ih_j}{\partial q_k}\right)\boldsymbol{e}_k\right]
$$

$$
\left.+\frac{2}{\hbar}\frac{\partial}{\partial q_j}\left(\frac{\hbar}{h_i}\frac{\overline{v_i'v_l'}}{h_l}\frac{1}{h_j}\frac{\partial h_j}{\partial q_l}\right)+\frac{2}{\hbar}\frac{\partial}{\partial q_i}\left(\frac{\hbar}{h_i}\frac{\overline{v_j'v_l'}}{h_l}\frac{1}{h_i}\frac{\partial h_i}{\partial q_l}\right)\right\}
$$

$$
-2\nu\left\{\overline{\frac{S_{jk}'}{h_k}\frac{\partial}{\partial q_k}\left(\frac{v_i'}{h_j}\right)}+\overline{\frac{S_{ik}'}{h_k}\frac{\partial}{\partial q_k}\left(\frac{v_j'}{h_i}\right)}+\overline{S_{kk}'\left(\frac{v_i'}{h_jh_k}\frac{\partial h_k}{\partial q_j}+\frac{v_i'}{h_ih_k}\frac{\partial h_k}{\partial q_i}\right)}\right\} \tag{11}
$$

在以上所有方程中，对 k，l 下标采取了 Einstein的约定记号，而对 i, j 则不是的.

四、代数模型

为了使方程封闭，需要进一步加上一些模型关系. 最简单的关于子格雷诺应力 $\overline{v_i'v_j'}$ 的模型为

$$
-\overline{v_i'v_j'}=\nu^*\overline{S}_{ij}+\frac{2}{3}\delta_{ij}\overline{E'} \tag{12}
$$

其中

$$
\nu^*=f(c\Delta)^2S；\quad S=\left\{\frac{1}{2}S_{kl}S_{kl}\right\}^{1/2}；\qquad \Delta=\{\hbar\Delta q_1\Delta q_2\Delta q_3\}^{1/3} \tag{13}
$$

c 为选用的常数，f 为一与大尺度量有关的数. 为确定 $\overline{E'}$，f 等，用以下代数模型:

根据Rotta[7]和Launder[8]的建议有

$$
\overline{\frac{p'}{\rho}\left(\frac{1}{h_i}\frac{\partial v_j'}{\partial q_i}+\frac{1}{h_j}\frac{\partial v_i'}{\partial q_j}\right)}=-c_1\frac{\varepsilon}{\overline{E'}}\left(\overline{v_i'v_j'}-\frac{2}{3}\delta_{ij}\overline{E'}\right)-\frac{c_2+8}{11}\left(P_{ij}\right.
$$

$$
\left.-\frac{2}{3}P\delta_{ij}\right)-\frac{30c_2-2}{55}\cdot 2\overline{E'}\,\overline{S}_{ij}-\frac{8c_2-2}{11}\left(D_{ij}-\frac{2}{3}P\delta_{ij}\right)
$$

$$
+\left\{\frac{1}{8}\frac{\varepsilon}{\overline{E'}}\left(\overline{v_i'v_j'}-\frac{2}{3}\delta_{ij}\overline{E'}\right)+\frac{3}{200}\left(P_{ij}-D_{ij}\right)\right\}\frac{\overline{E'}^{3/2}}{\varepsilon}\cdot f(x_2) \tag{14}
$$

其中x_2为与壁面的垂直距离，一般选为$f(x_2)=1/x_2$. 此外上式中

$$-P_{ij}=\frac{\overline{v_i'v_k'}}{h_k}\frac{\partial\bar{v}_j}{\partial q_k}+\frac{\overline{v_j'v_k'}}{h_k}\frac{\partial\bar{v}_i}{\partial q_k}-c_p\bar{v}_i\left(\frac{\overline{v_j'v_k'}}{h_jh_k}\frac{\partial h_j}{\partial q_k}-\frac{\overline{v_k'v_k'}}{h_jh_k}\frac{\partial h_k}{\partial q_j}\right)$$

$$-c_p\bar{v}_j\left(\frac{\overline{v_i'v_k'}}{h_ih_k}\frac{\partial h_i}{\partial q_k}-\frac{\overline{v_k'v_k'}}{h_jh_k}\frac{\partial h_k}{\partial q_j}\right)$$

$$-D_{ij}=\frac{\overline{v_i'v_k'}}{h_j}\frac{\partial\bar{v}_k}{\partial q_j}+\frac{\overline{v_j'v_k'}}{h_i}\frac{\partial\bar{v}_k}{\partial q_i}-c_p\bar{v}_i\left(\frac{\overline{v_j'v_k'}}{h_jh_k}\frac{\partial h_j}{\partial q_k}-\frac{\overline{v_k'v_k'}}{h_jh_k}\frac{\partial h_k}{\partial q_j}\right)$$

$$-c_p\bar{v}_j\left(\frac{\overline{v_i'v_k'}}{h_ih_k}\frac{\partial h_i}{\partial q_k}-\frac{\overline{v_k'v_k'}}{h_jh_k}\frac{\partial h_k}{\partial q_j}\right)$$

$$P=\frac{1}{2}P_{ll} \tag{15}$$

这里讨论的是大R_e数下、统计意义上准定常的湍流运动，故设$\frac{\partial\overline{v_i'v_j'}}{\partial t}\approx 0$，$\frac{\partial\overline{v_i'v_j'}}{\partial q_k}\approx 0$，$(i\neq j\neq k)$. 略去三阶小量$\overline{v_i'v_j'v_k'}$及压力和速度的脉动量的相关，又设

$$\varepsilon_{ij}=2\nu\left\{\overline{\frac{S'_{jk}}{h_k}\frac{\partial}{\partial q_k}\left(\frac{v_i'}{h_j}\right)}+\overline{\frac{S'_{ik}}{h_k}\frac{\partial}{\partial q_k}\left(\frac{v_j'}{h_i}\right)}+\overline{S'_{ik}\left(\frac{v_i'}{h_jh_k}\frac{\partial h_k}{\partial q_j}+\frac{v_j'}{h_ih_k}\frac{\partial h_k}{\partial q_i}\right)}\right\}=\frac{\overline{v_i'v_j'}}{\overline{E'}}\varepsilon\delta_{ij} \tag{16}$$

$$\varepsilon=c_E\frac{\overline{E'}^{3/2}}{\Delta} \tag{17}$$

将(12)--(17)代入(10),(11)可得方程

$$\overline{v_i'v_j'}\left(\frac{\bar{v}_k}{h_ih_k}\frac{\partial h_i}{\partial q_k}+\frac{\bar{v}_k}{h_jh_k}\frac{\partial h_j}{\partial q_k}+(c_1+\delta_{ij})c_E\overline{E'}^{1/2}/\Delta\right)+\overline{v_i'v_k'}\left(\frac{1}{h_k}\frac{\partial\bar{v}_j}{\partial q_k}+\frac{\bar{v}_j}{h_jh_k}\frac{\partial h_j}{\partial q_k}\right.$$

$$-2\frac{\bar{v}_k}{h_jh_k}\frac{\partial h_k}{\partial q_j}+\frac{c_2+8}{11}\left[\frac{1}{h_k}\frac{\partial\bar{v}_j}{\partial q_k}-\frac{c_p}{h_k}\frac{\bar{v}_j}{h_i}\frac{\partial h_i}{\partial q_k}\right]+\frac{8c_2-2}{11}\left[\frac{1}{h_j}\frac{\partial\bar{v}_k}{\partial q_j}-\frac{c_p\bar{v}_j}{h_ih_k}\frac{\partial h_i}{\partial q_k}\right]\Bigg)$$

$$+\overline{v_j'v_k'}\left(\frac{1}{h_k}\frac{\partial\bar{v}_i}{\partial q_k}+\frac{\bar{v}_i}{h_ih_k}\frac{\partial h_i}{\partial q_k}-2\frac{\bar{v}_k}{h_ih_k}\frac{\partial h_k}{\partial q_i}+\frac{c_2+8}{11}\left[\frac{1}{h_k}\frac{\partial\bar{v}_i}{\partial q_k}-\frac{c_p\bar{v}_i}{h_kh_j}\frac{\partial h_j}{\partial q_k}\right]\right.$$

$$+\frac{8c_2-2}{11}\left[\frac{1}{h_i}\frac{\partial\bar{v}_k}{\partial q_i}-\frac{c_p\bar{v}_i}{h_jh_k}\frac{\partial h_j}{\partial q_k}\right]\Bigg)+\overline{v_k'v_k'}\frac{9c_2+6}{11}\left(\frac{c_p}{h_k}\left[\frac{\bar{v}_i}{h_j}\frac{\partial h_k}{\partial q_j}+\frac{\bar{v}_j}{h_i}\frac{\partial h_k}{\partial q_i}\right]\right.$$

$$\left.-\frac{2}{3}\frac{c_p}{h_k}\frac{\bar{v}_l}{h_l}\frac{\partial h_k}{\partial q_l}\delta_{ij}\right)-\overline{v_l'v_k'}\frac{6c_2+4}{11}\frac{1}{h_k}\left(\frac{\partial\bar{v}_l}{\partial q_k}-\frac{c_p\bar{v}_l}{h_l}\frac{\partial h_l}{\partial q_k}\right)\delta_{ij}$$

$$=\frac{2}{3}c_1c_E\frac{\overline{E'}^{3/2}}{\Delta}\delta_{ij}-\frac{60c_2-4}{55}\overline{E'}\,\overline{S}_{ij}+\left\{\frac{c_E}{8}\frac{\overline{E'}^{1/2}}{\Delta}\left(\overline{v_i'v_j'}-\frac{2}{3}\delta_{ij}\overline{E'}\right)+\frac{3}{200}(P_{ij}\right.$$

$$\left.-D_{ij})\right\}\frac{\Delta}{c_E}f(x_2) \tag{18}$$

和

$$\overline{v_l'v_k'}\left(\frac{1}{h_k}\frac{\partial\bar{v}_l}{\partial q_k}-\frac{\bar{v}_l}{h_lh_k}\frac{\partial h_l}{\partial q_k}\right)+\overline{v_k'v_k'}\frac{\bar{v}_l}{h_lh_k}\frac{\partial h_k}{\partial q_l}+c_E\frac{\overline{E'}^{3/2}}{\Delta}=0 \tag{19}$$

若设

$$\overline{v_i'v_j'}=(\overline{v_i'v_j'})_{\rm iso}+(\overline{v_i'v_j'})_{\rm in} \tag{20}$$

其中$(\ )_{iso}$是各向同性部份，$(\ )_{in}$为各项异性部分．各向异性部份满足方程

$$\overline{v'_i v'_k}\left(\frac{1}{h_k}\frac{\partial \bar{v}_j}{\partial q_k}+\frac{\bar{v}_j}{h_j h_k}\frac{\partial h_j}{\partial q_k}-\frac{2}{h_j}\frac{\bar{v}_k}{h_k}\frac{\partial h_k}{\partial q_j}\right)+\overline{v'_j v'_k}\left(\frac{1}{h_k}\frac{\partial \bar{v}_i}{\partial q_k}+\frac{\bar{v}_i}{h_j h_k}\frac{\partial h_i}{\partial q_k}-\frac{2}{h_i}\frac{\bar{v}_k}{h_k}\frac{\partial h_k}{\partial q_i}\right)$$

$$=\left\{\frac{c_E}{8}\frac{\overline{E'}^{1/2}}{\Delta}\left(\overline{v'_i v'_j}-\frac{2}{3}\delta_{ij}\overline{E'}\right)+\frac{3}{200}(P_{ij}-D_{ij})\right\}\cdot\frac{\Delta}{c_E}f(x_2) \tag{21}$$

(18) 式中右端去掉$\{\ \ \}$项则得关于$(\overline{v'_i v'_j})_{iso}$的方程，可解得

$$(\overline{v'_i v'_j})_{iso}=F_{ij}(\bar{v}_1,\ \bar{v}_2,\ \bar{v}_3,\ \overline{E'}^{1/2}/\Delta) \tag{22}$$

将它们代入 (19) 式可得关于$\overline{E'}^{1/2}/\Delta$的方程

$$G(\bar{v}_1,\ \bar{v}_2,\ \bar{v}_3,\ \overline{E'}^{1/2}/\Delta)=0 \tag{23}$$

它得到的关于$\overline{E'}^{1/2}/\Delta$的最小非负根则为$\overline{E'}^{1/2}/\Delta$的值，再由

$$f=\frac{c}{S}\frac{\overline{E'}^{1/2}}{\Delta} \tag{24}$$

可以得到f值，代入(12)式得$(\overline{v'_i v'_j})_{iso}$．把$\overline{E'}$值代入(19)式可以解得$(\overline{v'_i v'_j})_{in}$值．由此可得

$$(\overline{v'_i v'_j})_{in}=\nu^{(ij)}\langle\overline{S_{ij}}\rangle,\quad(\nu^*)_{iso}=f(c\Delta)^2\overline{S} \tag{25}$$

$\langle S_{ij}\rangle$为S_{ij}的随时间的统计平均值．

五、直槽道内的湍流数值模拟

这里首先用上述模型计算直槽道内的湍流，其中几何尺寸及座标如图所示．这里计算$Re\to\infty$的情况，另外可以把压力分成二个部份:

$$p=p_0+p_1 \tag{26}$$

其中$dp_0/dx=\text{const}$．

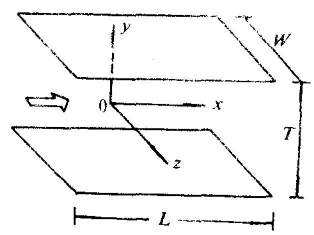

图 1

把(26)式代入(9)式，并将上述代数模型代入整理后可得如下方程

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0 \tag{27}$$

$$\left.\begin{aligned}
&\frac{\partial \bar{u}}{\partial t}+[\bar{v}]\frac{\partial \bar{u}}{\partial y}-\frac{\partial}{\partial y}\left([\nu^*]\frac{\partial \bar{u}}{\partial y}\right)+\frac{\partial \bar{p}_1}{\partial x}=H_x\\
&\frac{\partial \bar{v}}{\partial t}+[\bar{v}]\frac{\partial \bar{v}}{\partial y}-\frac{\partial}{\partial y}\left([\nu^*]\frac{\partial \bar{v}}{\partial y}\right)+\frac{\partial \bar{p}_1}{\partial y}=H_y\\
&\frac{\partial \bar{w}}{\partial t}+[\bar{v}]\frac{\partial \bar{w}}{\partial y}-\frac{\partial}{\partial y}\left([\nu^*]\frac{\partial \bar{w}}{\partial y}\right)+\frac{\partial \bar{p}_1}{\partial z}=H_z
\end{aligned}\right\} \tag{28}$$

其中[]表示在$y=\text{const}$平面上的平均值，H_x，H_y，H_z是右端项的简化记号．(27)(28) 式的边界条件为上下壁面上$\bar{u}=\bar{v}=\bar{w}=0$以及$\partial\bar{p}_1/\partial y=0$，在$x$及$z$方向上则有周期条件．

在计算中采用y方向变尺度网格，x和z方向则是等尺度的．$\bar{u}$，$\bar{v}$，$\bar{w}$，$\bar{p}$值定义在网格中央．y方向网格选用按公式

$$y_i=\frac{T}{2}\left\{\frac{1}{a}th\{(2\eta_i-1)th^{-1}a\}+1\right\},\quad \eta_i=\frac{i-1}{N};\quad i=1,2,\cdots\cdots N,N+1 \tag{29}$$

确定.

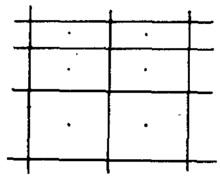

图 2

由于 x，z 方向为周期条件，所以取 $W=4T$，$L=8T$，并设 $\bar{u}$，$\bar{v}$，$\bar{w}$，$\bar{p}$ 都可以展为富氏级数，代入(27)、(28)式后得

$$il\frac{2\pi}{L}\hat{u}+\frac{\partial\hat{v}}{\partial y}+ik\frac{2\pi}{W}\hat{w}=0 \tag{30}$$

$$\frac{\partial\hat{v}_i}{\partial t}+[v]\frac{\partial\hat{v}_i}{\partial y}-\frac{\partial}{\partial y}\left([v^*]\frac{\partial\hat{v}_i}{\partial y}\right)+\begin{cases}i\frac{2\pi}{L}l\hat{p}_1\\ \frac{\partial\hat{p}_1}{\partial y}\\ i\frac{2\pi}{W}k\hat{p}_1\end{cases}=\begin{cases}\hat{H}_x\\ \hat{H}_y\\ \hat{H}_z\end{cases} \tag{31}$$

其中 $\hat{v}_i=\hat{u}$，$\hat{v}$，$\hat{w}$. 这里“∧”表示富氏展式的系数. 上二式可用Crank-Nickolson格式方便地求解，再作逆变换即得 $\bar{u}$，$\bar{v}$，$\bar{w}$，p 的值.

为保证计算的稳定性，时间步长选用方法如下：

$$\Delta t=\frac{f_k}{\frac{|u|_{max}}{\Delta x}+\frac{|w|_{max}}{\Delta z}+2v^*_{max}\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)} \tag{32}$$

其中 f_k 为某一常数，这里选用 $f_k=0.2$—0.5.

模型中各常数的选用如表1所示：

表 1

常 数	c_f	c_p	c_1	c_2	c	a
公 式 号	(17)	(15)	(14)	(14)	(13)	(29)
数 值	1.10	0.3	2.5	0.75	1.5	0.9

为节省计算时间，$\overline{E'}^{1/2}/\Delta$ 并非每步都计算，而是每10步计算一次.

网格选用32×16×16，在M—150上进行计算，每步机时为6分钟，计算进行了500步，耗机时50小时. 计算结果见附图3—图7.

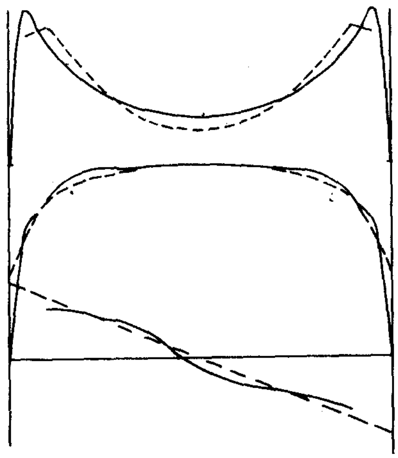

图 3

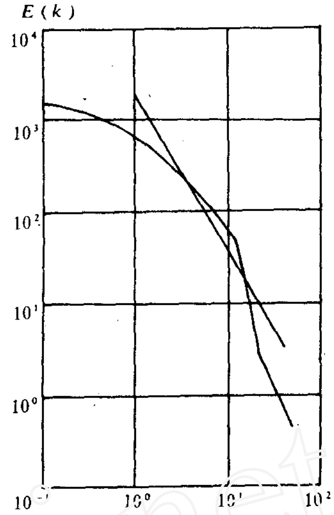

图 4

图 5　等E'线

图 6　等p'线

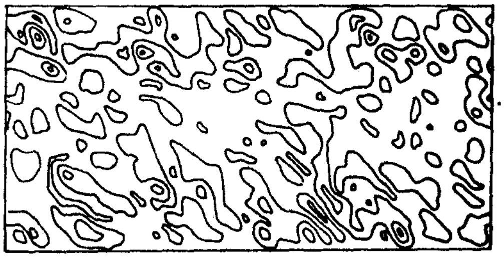

图 7　等u'线

六、结　论

用代数模型作为大涡模拟计算中湍流模型对于 Re 数很大的情况是可用的. 计算结果和Schumann的比较还是比较好的. 该模型将进一步用来计算弯曲通道内的湍流计算.

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ALGEBRAIC MODELLING OF LARGE EDDY SIMULATION

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ABSTRACT In the present paper an algebraic modelling is developed, which is used to calculate the isotropic and inhomogeneous parts of quasi-Reynolds' stresses. This algebraic modelling is used as a model in large eddy simulation. The turbulent flow in a straight channel was numerically simulated with LES of algebraic modelling. The primary results show that this modelling is available and hopeful.

KEY WORDS turbulence, large eddy simulation, computational fluid dynamics.