含有微观缺陷的弹性材料的统计理论

周叔昂*　　谢庆祥

(瑞典皇家理工学院力学系)

摘　　要

工程材料或多或少地含有某些缺陷，如位错．夹杂物，裂纹等等。这些缺陷的存在，往往严重地影响材料的物理性质。关于缺陷对材料机械性质的影响的专门研究可以追溯到1934年 Taylor 关于位错的工作[1]，1961 年 Eshelby 处理非均匀夹质物的巧妙方法[2]，1960 年 Bristow 关于含有微观裂纹的弹性体的研究[3]以及 1962 年 Hashin 和 Shtrikman 处理非均匀弹性材料的变分原理[4]。

我们知道这些非统计的方法确实能对某些含有缺陷的材料的有效机械性质给出合理的估计，但是在另外一些情形它们却不适用。这是由于在这些方法中所使用的缺陷的周期分布或均匀随机分布的假定并不真实地反映了材料的微观结构。因此有必要研究某种能考虑到材料微观结构的模型。此外，在实际材料中，不同类型的缺陷往往同时存在。可是以往处理不同的缺陷却要使用不同的方法。因此寻找出一种统一处理的方法可以使问题得以简化。

本文的目的就是要试图解决这两个问题。使用由 Siems[5]，Kovács[6]以及 Zhou 和 Hsieh[7,8]等发展起来的模拟缺陷的弹性多极子方法，对应地我们发展一个基于微观力学分析的统计理论。这个理论使我们有可能在缺陷的离散微观结构与宏观的连续近似之间建立起一座桥梁，并且可以使我们从一个共同的基础上导出各种类型的缺陷体的宏观性质。

作为这个理论的具体应用，文中将给出两个例子。前一个例子是研究弹性体中微观裂纹群与一个宏观裂纹相互作用的问题。在这个问题中，由于宏观裂纹的存在，使得在裂纹尖端处的应力场发生急剧的变化。因此通常用于处理微观裂纹群的均匀有效应力场的假定（Hill[9]，Budiansky[10]）不再适用。1983 年 Chudnovsky 和 Kachanov[11]注意到了这个事实。他们用一个推广了的自洽方法研究了二维的宏观裂纹与微观裂纹群的相互作用问题。Zhou 和 Hsieh[12]则考虑了三维宏观裂纹与微观缺陷群的相互作用问题。可是，正如对一切离散的微观结构系统进行分析所导致的结果一样，这个适用于非均匀有效应力场的广义自洽方法给出了一组用于确定大量的微观缺陷“中心”处的未知的有效应力的线性代数方程，尽管原则上可以求解这组方程。运用本文所发展的弹性多极子统计理论，这组离散的线性代数方程可以被简化成一个用于确定平均有效应力场的自洽的线性积分方程。这个积分方程可以较容易地被求解。文中给出的第二个例子是关于确定含有非均匀随机分布的微观缺陷的弹性体的有效弹性模量。

* 暂离上海复旦大学应用力学系

A Statistical Theory of Elastic Materials with Micro-defects

S. A. Zhou and R. K. T. Hsieh*

(Department of Mechanics Royal Institute of Technology S-100 44, Stockholm Sweden)

Abstract

Engineering materials always contains defects, such as dislocations, inclusions, cracks···, and many material physical properties are considerably changed by their presence. Specific investigations of their effects on the mechanical properties of materials can be backdated to Taylor's work on dislocations (1934 [1]), Eshelby's elegant method on inclusions (1961 [2]), Bristow's study on microcracked solids (1960 [3]), and Hashin and Shtrikman's powerful variational principle on inhomogeneous materials (1962 [4]).

It is known, however, that those non-statistical approaches do give reasonable estimates for the effective mechanical properties for some defectuous materials but they fail for others, This is due to the fact that the periodic defect array model or random model used in these approaches does not accurately represent the microstructure of the material. Some improved models accounting for the material microstructure are therefore needed. In real materials, too the different types of defects often co-exist. However, each of them was treated in a different way. It would be therefore convenient to find a unified approach for the treatment of different types of defects.

The purposes of this paper are concerned with the answering to these two problems. Using the approach of elastic multipole modeling of defects originated by Siems [5], Kovács [6], Zhon and Hsieh [7, 8], we develop accordingly a statistical theory based on micromechanics analysis which would make it possible to set up a bridge between the macroscopic continuum approximation and the discrete microstructure of micro-defects, and to derive the macro-properties of all types of solid defects from a common basis.

Two illustrations are given. The first one is concerned with the study of the interaction of a macrocrack with distributed microcracks in an elastic solid.

In this problem, the usual assumption of a uniform effective field for the treatment of cracked solids (see Hill [9] and Budiansky [10]) may be not suitable

* On leave from the Department of Applied Mechanics, Fudan University, Shanghai, China

since the existence of the macrocrack in a microcracked solid makes the stress field near the macrocrack tip varies rapidly. This has been noticed by Chudnovsky and Kachanov (1983 [11]) who studied the two-dimensional interaction of a crack with a field of microcracks distributed in the vicinity of the main crack, and by Zhon and Hsieh (1984 [12]) who gave the results of three-dimensional interaction of a crack with a field of micro-defects (including microcracks) in an elastic solid. These considerations based on a generalized self-consistent method, in which the effective stress field may be non-uniform, however arrived at a system of linear algebraic equations for the determination of large number of unknown-values of effective stress fields at the defect "centers" although they can be solved exactly in principle. Through this statistical theory, an attempt is made to reduce such a phenomenon resulting from the analysis of any discrete microstructure system to an appropriate macroscopic continuum approximation. In the case of the interaction problem, the system of discrete linear algebraic equations is reduced to a self-consistent linear integral equation for the determination of the average effective stress field, which may be solved more easily.

The second application is concerned with the determination of effective elastic moduli of solids with micro-defects which may be statistically inhomogeneous.

REFERENCES

[1] Taylor, G. I., Proc. Roy. Soc., A145 (1934), 362.

[2] Eshelby, J. D., Progress in Solid Mech., 2 (ed. by I. Sneddon & R. Hill), North Holland, Amsterdam (1961), 89.

[3] Bristow, J. R., *Bri. J. Appl. Phys.*, 11 (1960), 81.

[4] Hashin, Z., Shtrikman, S., *J. Mech. Phys. Solids*, 10 (1962), 335.

[5] Siems, R., *Phys. Status Solidi*, 30 (1968), 645.

[6] Kovács, I., *Physica*, 94B (1978), 177.

[7] Zhou, S. A., Stationary Multipole Inhomogeneous Inclusions in Elastic Media, to appear in *J. Tech. Phys.*

[8] Zhou, S. A. and Hsieh, R. K. T., Elastic Monopole Modeling of Defects in Thermoelastic Media (forthcoming).

[9] Hill, R., A Self-Consistent Mechanics of Composite Material, *J. Mech. Phys. Solids*, 13 (1965), 213.

[10] Budiansky, B. and O'Connell, R. J., *Int. J. Solids Structures*, 12 (1976), 81.

[11] Chudnovsky, A. and Kachanov, M., *Lett. Appl. Engng. Sci.*, 21 (1983), 1009.

[12] Zhou, S. A. and Hsieh, R. K. T., Elastic Monopole Modeling of the Interaction of a Crack with Microcracks and Lattice Defects (forthcoming).