关于敏感动力过程以及从表观决定性过程向完全随机过程的过渡

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摘　要*

本文对掷硬币的动力学过程作了详细分析。通过计算，指出结果究竟如何以及为什么极其敏感地依赖于初始条件。结果也表明，随着硬币质心高度的增加，最后方位（"正面"或"反面"）变得对初始条件（初始速度、角速度，初始方位）、空气阻力系数，以及"桌面"的能量吸收因子越来越敏感。如果保持"正面初始向上，而允许其它初始参数有一个小的变化范围，那么，当质心初始高度 h 充分小时，最终方位为"正面"的频率为1，当 h 充分大时，这个频率将近于 1/2。一个有趣的问题是，当 h 增加时，这个频率怎样从 1 连续地过渡到近于 1/2。详细的计算表明，这一"过渡"与层流到湍流的"过渡"有许多相似之处。本文指出了"过渡阶段"和"完全随机阶段"的一个基本区别：在"完全随机阶段"，决定性的单过程对初始条件和动力学参数极端敏感，但系综的统计性质对于初始条件和动力学参数小的变化并不敏感。与此相反，在"过渡阶段"，无论决定性的单过程，还是系综的统计性质，都敏感地依赖于初始条件和动力学参数。造成过渡阶段这一特点的机制是在参数空间存在"长鍊结构。"本文并讨论了这一分析对其它随机现象可能具有的启示。

On the Sensitive Dynamical System and the Transition from the Apparently Deterministic Process to the Completely Random Process

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Abstract*

The detailed analysis of the dynamical process of coin tossing is made. Through calculations, it is illustrated how and why the result is extremely sensitive to the initial conditions. It is also shown that, as the initial height of the mass center of the coin increases, the final configuration, i.e. "head" or "tail", becomes more and more sensitive to the initial parameters (the initial velocity and angular velocity, the initial orientation), the coefficient of the air drag, and the energy absorption factor of the surface on which the coin bounces. If we keep the "head" upward initially but allow a small range for the change of some other initial parameters, the frequency that the final configuration is "head" would be 1 if the initial height h of the mass center is sufficiently small, and would be close to 1/2 if h is sufficiently large. An interesting question is how this frequency changes continuously from 1 to 1/2 as h increases. Detailed calculations show that such a "transition" is very similar to the transition from laminar to turbulent flows. A basic difference between the "transition stage" and the "completely random stage" is indicated. In the "completely random stage", the deterministic process of the individual case is extremely sensitive to the initial conditions and the dynamical parameters, but the statistical properties of the ensemble are insensitive to the small changes of the initial conditions and the dynamical parameters. On the contrary, in the "transition stage", both the deterministic process of the individual case and the statistical properties of the ensemble are to the initial conditions and the dynamical parameters. The mechanism for this feature of the "transition stage" is the existence of the "long-train structure" in the parameter space. The implications of this analysis on some other random phenomena are discussed.